Measurement of display transfer characteristic (gamma, γ)

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EBU Sub–group G4 (Video origination equipment) has recently completed a new EBU publication setting out recommended procedures for the measurement of the colorimetry of studio colour monitors. This EBU TUTORIAL, prepared by a member of Sub–group G4, presents the subject in rather more detail, especially as regards the mathematical treatment of measurement results, and gives a worked example.

1. Introduction

The accurate measurement of display transfer characteristics is notoriously difficult and it requires skill and precision to produce repeatable results. Quite apart from the problems resulting from the use of insufficiently–precise measuring instruments, errors are usually present in measurement data as a result of inaccurate setting of the display black level. These errors can give rise to false conclusions about the numerical value of the power law involved (gamma), or even whether there is a numerical value to be found. Two measurement methods have been developed recently and these largely remove the uncertainty resulting from inaccurate measurements [1].

In order to understand these methods, it is necessary to consider first the nature of curves which involve power laws.

2. Power–law curves (gamma)

It is widely believed that the transfer characteristic of a typical cathode–ray tube has a power law of 2.8 ± 0.3. However, let us consider the case of a cathode–ray tube with a perfect power law of 2.8, i.e. $L = V^{2.8}$ where L is measured in units of light and V is the cathode–ray tube drive voltage. For studio monitors, L and V usually vary over a range from zero to 80 cd/m^2 and zero to 700 mV respectively and thus the transfer characteristic can be defined as in equation (1).

$$L = 80 \left(\frac{V}{0.7}\right)^{2.8} \tag{1}$$

This relationship is defined by the physical properties of the cathode–ray tube and should therefore be valid over the entire dynamic range of the cathode–ray tube drive signals. However, to be realistic, let

us assume that the signal range is 40 dB, corresponding to a voltage–contrast of 100:1. In this case, V varies between 700 mV and 7 mV and L correspondingly varies from 80 cd/m^2 to 0.0002 cd/m^2, a light–contrast of 398,107:1. This characteristic is shown in *Fig. 1*, plotted on logarithmic scales, and has a slope of 2.8, i.e. the value of gamma (γ).

In practice, there are always offset errors in either the drive voltage or the measurement of light, or both, and so the general equation (2) for the transfer characteristic is more complex.

$$L = l_o + 80 \left(\frac{V - v_o}{0.7} \right)^\gamma \qquad (2)$$

The offsets in this equation are v_o, the voltage error in setting the black level and l_o, the combined effect of black–level setting error, stray light, and measurement instrument offset. It is routine practice to ignore both of these offsets; indeed, the black level is usually set to a high level so that it can be measured with precision.

Black level is frequently set as high as 0.5 cd/m^2 when measuring display contrast ratio, but this is entirely unsuitable for measuring the transfer characteristic, since the available voltage–contrast range is reduced to only 6.1:1 on a display with γ of 2.8 (this is because $(0.5/80)^{1/2.8} = 1/6.1$). This characteristic is shown in *Fig. 2*, plotted logarithmically to the same scales as were used for *Fig. 1*, with a light offset ($l_o = 0.5$ cd/m^2). The straight line running through only the top two points has a slope of 2.77, whereas the best–fitting line through *all* the points has a slope of only 1.0097; the true power law is very much concealed by the presence of the offset. The curvature of the data points is severe and, at first sight, might indicate that no power law equation is involved at all. However, the curve is asymptotic to the correct slope of 2.8 at high values and it is tempting to discard the lower values and use only those extending down to, say, 12 dB. This would be a mistake, since there is useful information in *all* the data points and we should try to extract this information for use in analysis.

Fig. 3 shows the same power law again, this time with offsets in both L ($l_o = 0.003$) and V ($v_o = 0.007$), these values being representative of the magnitudes of offsets present in actual measurements. Clearly, the situation is now much more complex. There is curvature of the data points in opposing directions and it is very difficult to determine which points should be used to establish the power law. In fact, the slope through the top two points alone is 2.83 (greater than γ), the slope through all the points is 2.73 (less than γ), and that through the central range of points, i.e. from –24 dB to –6 dB, is 2.94 (much greater than γ). Traditionally, the top and bottom parts of the curve would be discarded and γ extracted from the central data points, giving the erroneous value of 2.94. It is tempting, when presented with such data, to conclude that the lower points (–30 dB and below) deviate from the power law because "the power law does not apply near cut–off", and that it does not apply at high levels (–4 dB and above) because of "phosphor saturation". In fact, neither
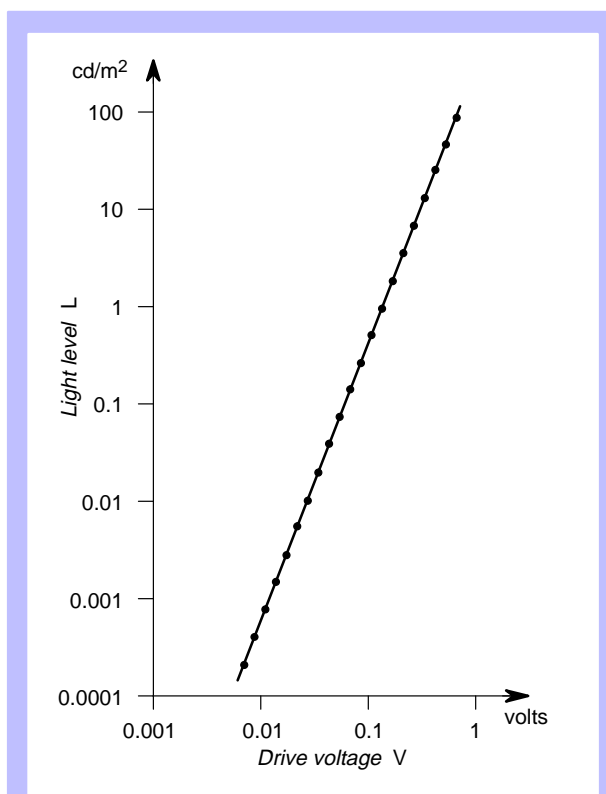

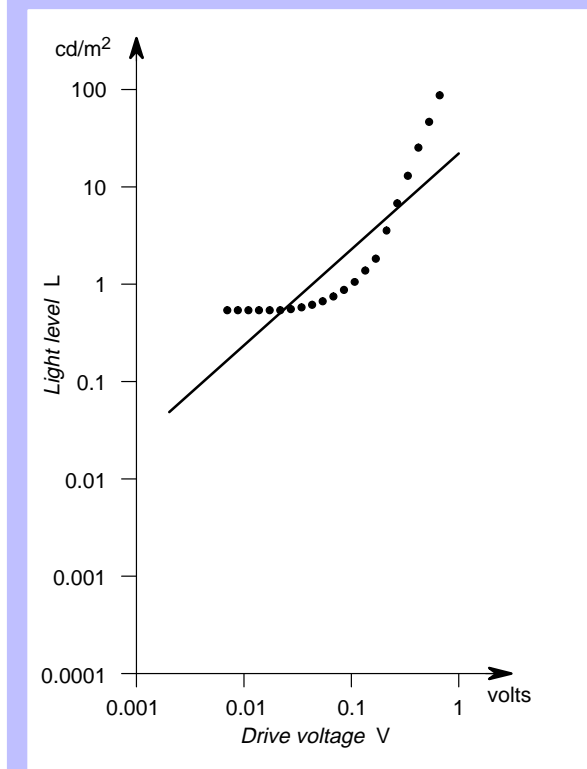Fig. 1 – Logartithmic plot of power law 2.8.

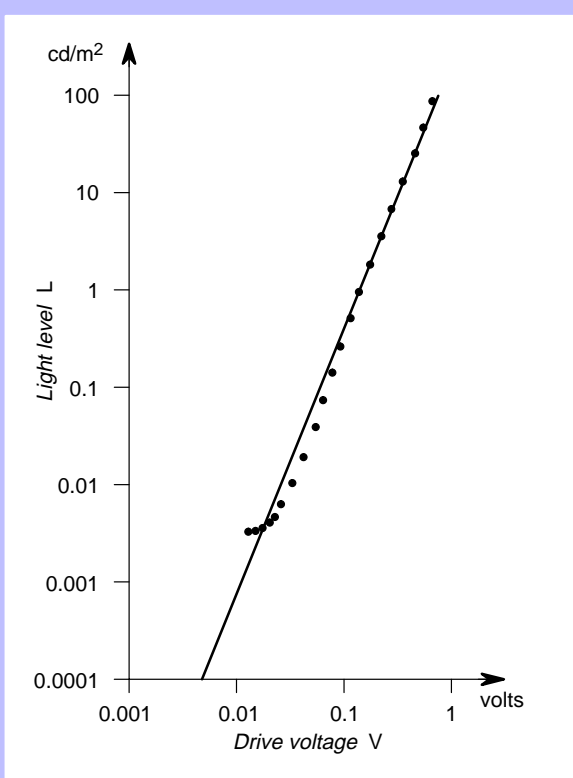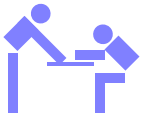Fig. 2 – Plot of power law 2.8 with offset of 0.5 in *L*.

Fig. 3 – Logartithmic plot of power law 2.8 with offsets of 0.5 in both *L* and *V*.

explanation is required since the curvature arises solely from the presence of offsets in the measurement data.

Evidently, what we need is a means of extracting the true value of γ from all the data points in the presence of these offsets, or a means of eliminating them from the measured data. Fortunately, such means are available.

3. Traditional measurement method

The simplest way of obtaining a set of data values describing the transfer characteristic is to make a series of individual measurements of brightness at regular intervals of drive voltage, using a calibrated attenuator to step the voltage over the desired range. This has the disadvantage that the photometer used to measure the light level must have very linear characteristics over a very large range; for a display with γ of 2.8 and voltage contrast of 100:1, the light–contrast will be $100^{2.8}$:1 (about 400,000:1). Photomultiplier tubes do achieve this degree of linearity and dynamic range and they are entirely adequate for these measurements when used with an auto–ranging digital volt–meter with sufficient range and precision. Alternatively, calibrated neutral–density filters can be interposed between the display and the photometer, and the characteristic is then derived from the measured drive voltage needed to produce the attenuated light for a large range of filter densities. This has two disadvantages: it requires a number of calibrated filters and it needs a very sensitive photometer which will always be operated at its highest sensitivity setting and thus could be noisy. Either method can produce a data table relating drive voltage to light output.

Measurement using either technique is a slow and labour–intensive operation, suited best to skilled operators. Also, as already seen, the data points will probably exhibit some curvature when plotted logarithmically, due to offset errors in the measurements. These errors can be removed, or at least estimated, mathematically, provided that both offsets are constant for the whole duration of the measurement session, and this is the basis of the method of extracting a value for γ.

Offset error removal by differential slope analysis

In *Section 2*, the transfer characteristic was assumed to be of the form shown in equation (2). If the measurements were taken on a stable display in controlled conditions, then the offsets l_o and v_o will both be numerically constant for the duration of the measurement session and they can each be removed separately from the equation by differentiation; for example, l_o is removed in equation (3).

$$\frac{dL}{dV} = 80\gamma \left(\frac{V - v_o}{0.7}\right)^{\gamma - 1} \tag{3}$$

The exponent in this equation, and thus the slope of the line running through the data points on a logarithmic plot, is $\gamma - 1$.

Now equation (3) can be simplified into the form of expression (4), where dL/dV is the slope of the data points taken a pair at a time.

$$\frac{dL}{dV} \propto (V - v_o)^{\gamma - 1} \tag{4}$$

For simplicity, dL/dV is defined in equation (5) where the subscripts n refer to the nth values in the measurement data table.

$$\frac{dL}{dV_n} = \frac{L_n - L_{n-1}}{V_n - V_{n-1}} \tag{5}$$

Each value of dL/dV is assumed to be relevant to a point midway between each adjacent pair. By simply plotting values of dL/dV versus V, one cause of data curvature is eliminated; this is shown in *Fig. 4*, for the same data values as were used in *Fig. 3*. The points curve monotonically since there is only one unknown offset, and this can be estimated by iterative estimation and inspection or by computer optimisation. With either method, a computer is needed. If manual estimation is chosen, the operator makes a guess at the value of v_o, and uses the computer to re–plot the data values, after which the operator again inspects the resultant curvature. After several iterations of such guessing, a value of v_o can usually be found which reduces the curvature to an acceptably low level. This process requires some operator skill and is time–consuming. A computer optimisation routine can automate this process of estimation and re–plotting, but a measure is needed for quantification of curvature of the data points about the linear–regression line. A flow diagram of a simple optimisation routine designed for this application, which includes a curvature quantification algorithm, is shown in the *Appendix*.

By either of these methods, a value for gamma (γ) can be found from the plot of equation (4), after optimising a value for v_o. Similarly, the value for v_o can be eliminated from the equations by substitution, leaving l_o as a constant. Equation (2) can be manipulated to provide an expression for $V–v_o$ which, when substituted into equation (3), gives equation (6) from which v_o has disappeared but l_o is present.

$$\frac{dL}{dV} = 80\gamma \left(\frac{L - l_o}{80}\right)^{\frac{\gamma - 1}{\gamma}} \tag{6}$$

This can be simplified into a proportionality as shown in expression (7), and the resultant curve for the original data is shown in *Fig. 5*.

$$\frac{dL}{dV} \propto (L - l_o)^{\frac{\gamma - 1}{\gamma}} \tag{7}$$

Again, there is only one source of data curvature, and the same mathematical process can be used to estimate the offset value l_o, either by hand or by computer optimisation, and a new value for γ can be found from the slope of this graph.

Ideally, the two values of γ thus found should be identical, but this will usually not be the case because of residual errors, such as the rounding of data values and the assumption that the offsets are constant. It is good practice to remove the estimated or calculated offsets l_o and v_o from the original data values and then to re–plot
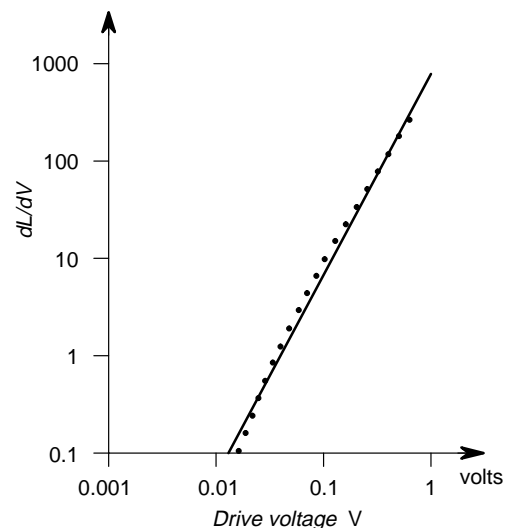

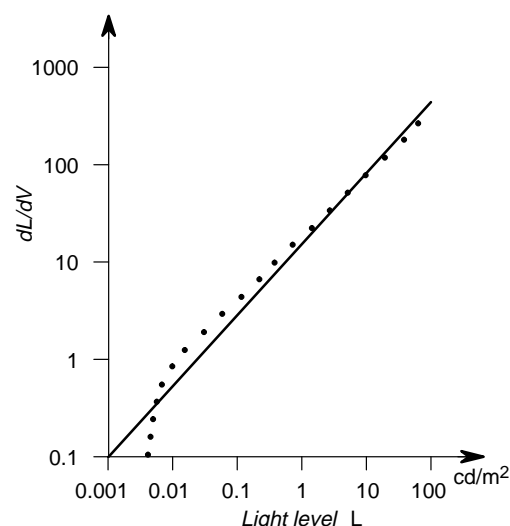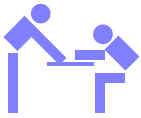Fig. 4 – Plot of dL/dV versus V for power law 2.8, with offsets in both L and V.

Fig. 5 – Plot of dL/dV versus L for power law 2.8, with offsets in both L and V.
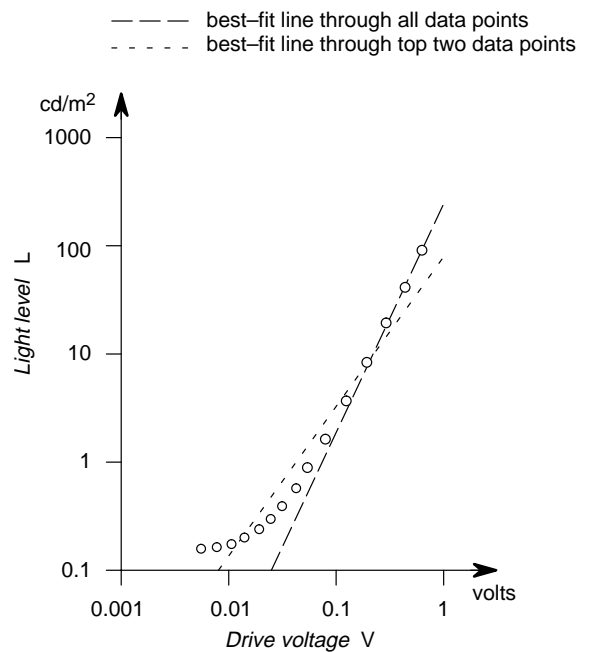
these revised values logarithmically; a third value for γ can be extracted from the slope of this new plot. This process provides good confirmation of the validity of the original data values, since the effects of noise and rounding errors are usually evident only at the lowest levels. The three values for γ can then be averaged to provide a realistic value.

A worked example of this process is given below, for a real set of measurements of one cathode–ray tube.

Worked example of differential slope analysis

V (dB)	V (volts)	L (cd/m^2)
0	0.777	104
−3	0.4956	50
−6	0.3508	24.4
−9	0.2484	12.0
−12	0.1758	6.07
−15	0.1245	3.16
−18	0.08812	1.73
−21	0.06239	1.00
−24	0.04417	0.614
−27	0.03127	0.409
−30	0.02214	0.288
−33	0.01567	0.221
−36	0.01109	0.181
−39	0.007854	0.154
−42	0.005600	0.137

— — — best–fit line through all data points
- - - - best–fit line through top two data points

1 Table of measurement values for a cathode ray tube display and a graph showing these values plotted logarithmically. The curvature of these data points is clearly seen.

V$_{mean}$ (volts)	L$_{mean}$ (cd/m^2)	Slope dL/dV
0.597781	77	264.14
0.423197	37.2	176.88
0.299600	18.2	121.02
0.212101	9.035	81.751
0.150156	4.615	56.667
0.106302	2.445	39.335
0.075256	1.365	28.364
0.053277	0.807	21.285
0.037717	0.512	15.893
0.026702	0.349	13.250
0.018904	0.255	10.364
0.013383	0.201	8.7397
0.009474	0.167	8.3330
0.006707	0.146	7.4112

2 Differential calculations derived from the values in Step 1.
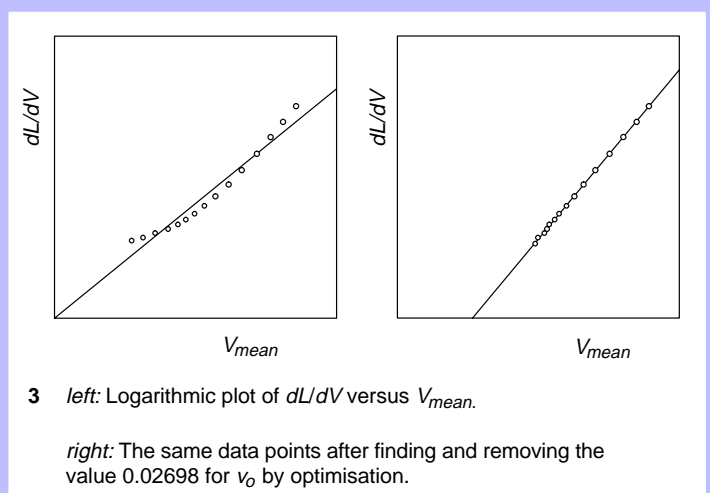
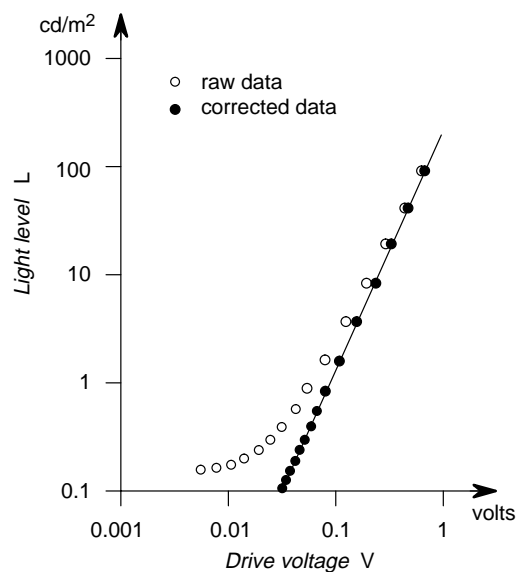**3** *left:* Logarithmic plot of *dL/dV* versus V$_{mean}$.

right: The same data points after finding and removing the value 0.02698 for v_o by optimisation.
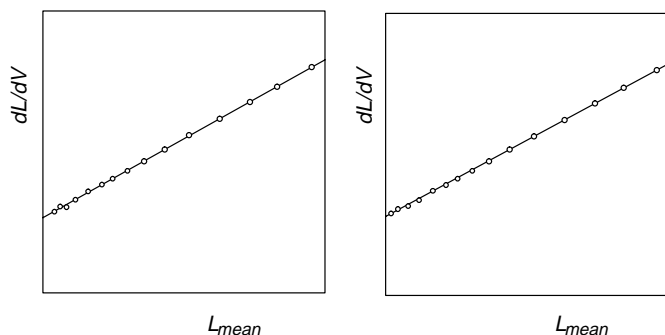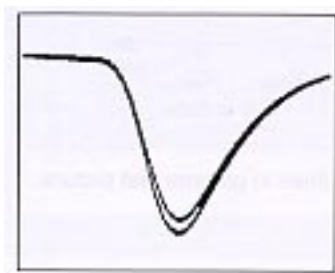
4 By a process similar to that in Step 3, this step shows:
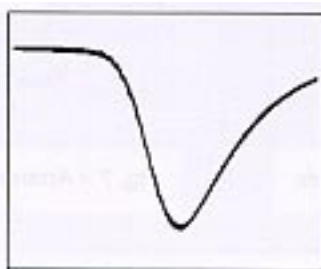
left: A plot of dL/dV versus L_{mean},

right: The same data points after finding and removing the value 0.034432 for l_o by optimisation.
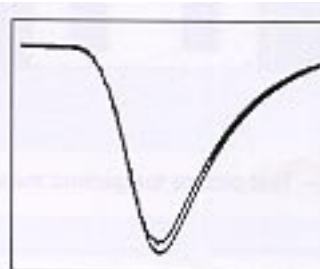

5 The original points re–plotted both before and after removal of l_o and v_o.

| $\gamma = 2.0$ | $\gamma = 2.2$ | $\gamma = 2.4$ |
| (separate traces) | (traces merged) | (separate traces) |

6 Waveforms obtained using the test–picture method of gamma measurement (see *Section 4* of the main text).

Field–rate waveform for the optimum patch and for two patches either side, at $\gamma + 0.2$ and $\gamma - 0.2$, filtered with a time–constant of about 1 ms.

Plot	Offset l_o	Offset v_o	γ
L vs V	none	none	1.405
dL/dV vs V	n/a	0.026981	2.232
dL/dV vs L	−0.034432	n/a	2.219
L vs V	−0.034432	0.026981	2.230
Test picture (see *Step 6*)	n/a	n/a	2.2

7 Calculated value of γ obtained by the best–fit algorithm (see *Appendix*) for each of these plots.

For comparison, the Table also contains the value of γ found by observing the test picture (Step 6).

The mean of the three values for γ found by the differential slope analysis method is 2.227, and this value agrees well with that found by observation of the test picture.
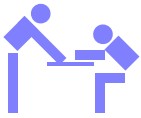
4. Gamma measurement using a test picture

A test signal has been devised which permits rapid and accurate measurement of γ, provided that such a power law exists for the display device in question. The signal, shown in *Fig. 6*, comprises ten test patches, each prepared for a different value of display γ, plus a peak white patch and two patches near black ($\pm 2\%$) which are used to set the white and black levels. Each of the ten test patches contains lines of three brightnesses, 25% are black, 25% are white and 50% have levels which are calculated to produce mid–grey when seen on a display with a different value of γ. The disposition of the scanning lines is shown in *Fig. 7*, where odd–numbered lines (comprising one field) are grey and even–numbered lines (the other field) are alternately black and white.
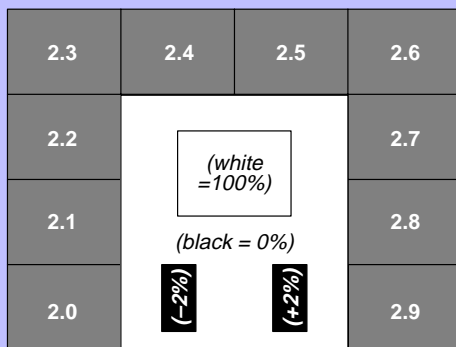

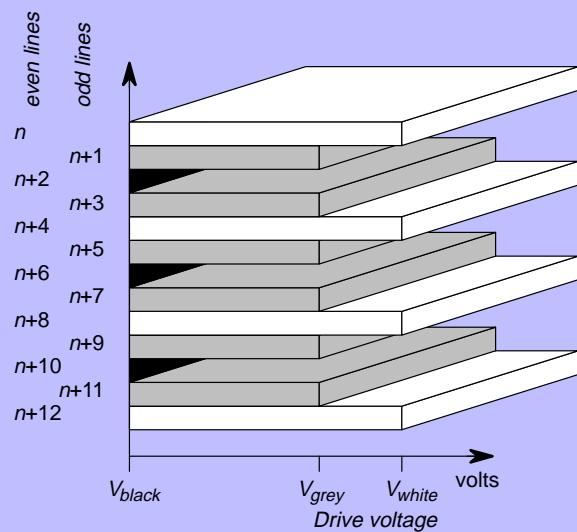Fig. 6 – Test picture for gamma measurements.

Fig. 7 – Arrangement of lines in gamma test picture.

When viewed from a suitable distance, the eye integrates the black and white lines to form a true mid–grey. Any brightness difference between the field of grey lines and the field of black and white lines is seen as flicker at picture–rate (25 Hz). The observer looks at the display of this test signal and can identify the test patch which exhibits minimum picture–rate (25 Hz) brightness flicker, and this indicates the value of γ for the display.

The test patch which produces no picture–rate flicker has the same brightness in each field. The voltage V_{grey}, which produced the mid–grey lines for that patch, must have been calculated using the true value of γ for the display. This condition is expressed in equation (8).

$$\frac{V_{grey}}{V_{white} - V_{black}} = 0.5^{\frac{1}{\gamma}} \tag{8}$$

The values of V_{grey} for the test signal are shown in *Table 1*, for values of V_{white} equal to 700 mV and V_{black} equal to 0 mV. In practical terms, only three points on the transfer characteristic are assessed: black, white and mid–grey; misleading results can arise if there is no genuine power–law relationship for the display device user test, e.g. for liquid–crystal and other solid–state displays. However, the method works well for displays which do have a power–law relationship. It is valid to assume that V_{black} is zero, since the test

γ	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
V_{grey} (mV)	495.0	503.2	510.8	517.9	524.4	530.5	536.2	541.5	546.5	551.2	555.6

Table 1
Mid–grey levels for a range of values of γ.

signal includes two patches near black to ensure accurate setting of the black level. Thus, the maximum possible error in black voltage level setting is $\pm 2\%$, which equates to a maximum error in black light–level of $\pm 0.0175\%$ of the peak brightness for a display with a γ of 2.8; this error is insignificant.

Inexperienced operators might have some difficulty in establishing which of the test patches exhibits least flicker. It is worth noting that the brightness difference of the mid–grey components of the patches adjacent to the optimum is only about $0.75\Delta L^*$, so that even experienced operators may not identify this patch accurately unless sufficient care is given. It is easier to identify the patch with minimum flicker by fixing the gaze on another part of the picture, for example the central white patch, and using the extra flicker–sensitivity of peripheral vision to look for flicker.

An alternative way of using the test signal is to use a photometer as a flicker detector. A photo–multiplier tube is suitable since it will have good temporal response, but it must be fitted with a low–pass filter with a time–constant of about 1 ms to remove line–frequency components. The detector is used to examine each patch in turn and its output is viewed on an oscilloscope scanning at field–rate. The traces from each field will be directly proportional to the mean brightness; hence the patch with minimum flicker can be identified as the one which produces two identical field traces. An example of the output of such a detector is shown in Step 6 of the worked example on *pages 36–37*, where the centre traces are from the optimum patch, and the traces on either side are those obtained from the patches calculated for the next–higher and next–lower values of γ. It is easier to identify the optimum patch in this way, although the differences between adjacent patches are quite small; the difference between the grey–level signals in adjacent patches is only 2.5 % but this is detectable if the photo–multiplier signal is reasonably noise–free.

The test picture is quite an undemanding video signal. It contains no essential high frequencies and can be distributed satisfactorily, even on VHS tape. Since the assessment is made on a static picture, the display does not need to be particularly stable, so domestic displays can be measured with good reliability. It could be useful in studio control rooms, as a means of ensuring that all displays have the same characteristic, since small adjustments of the black level can change the perceived value of γ by up to ± 0.2.

5. Conclusions

Two methods of extracting a value for the inherent power law in the transfer characteristics of cathode–ray tubes are now available. Both methods are reliable and give results which are consistently similar to each other. The analytical method is exhaustive and time–consuming but very precise, whilst the visual method is non–rigorous but produces acceptable results. Neither method is suited to displays which do not have a power–law relationship and no rigorous method is available yet for the measurement of such displays.

Bibliography

[1] EBU document Tech. 3273: **Methods of measurement of the colorimetric performance of studio monitors**

Appendix – Computer algorithm flow diagram for differential slope analysis

Measurement data are in arrays *L(m)* and *V(m)*   (*m* = 0 to *n*)

Differentiate arrays to obtain slope array *dL/dV(m)* (*m* = 0 to *n*–1)

Set *offset* = zero

Assign *dL/dV* array to *y* array   (0 to *n*–1)
Assign *L* or *V* array to *x* array   (0 to *n*–1)
Set *curl* = zero

Set variables: *a, b, c, d, e, f, g* = zero
Set *curl1* = *curl*

Trend line analysis loop: (*m* = 0 to *n*–1)
$x = LN(x(m) + offset)$: $y = LN(y(m))$
$a = a + x$: $b = b + y$: $c = c + xy$
$d = d + x{\wedge}2$: $e = e + y{\wedge}2$

$f = c - (ab)/(n{-}2)$: $g = d - (a{\wedge}2)/(n{-}2)$
$yo = (b - af/g)/(n{-}2)$: $slope = f/g$
Set *curl* = zero for curvature analysis

Curvature analysis loop: *m* = 0 to *n*–1
$error = y(m) - (offset + EXP\ y_o\ x{\wedge}slope)$
If *error* > 0 : *s* = +1 : else *s* = –1
If *m* is in either outer quartile : $curl = curl - s\ error{\wedge}2$
else : $curl = curl + s\ error{\wedge}2$

Curvature test : compare *curl* with *curl1*
increased?   same?   reduced?

reduce *offset*

increase *offset*

Optimisation completed;   results in *slope* and *offset*

If *dL/dV* versus *V* optimisation: *gamma* = 1 + *slope* : v_o = *offset*
If *dL/dV* versus *L* optimisation: *gamma* = 1/(1–*slope*) : l_o = *offset*