The auctions of UMTS spectrum in Europe have raised worldwide awareness of the potential value of this natural resource which, over the past decades, has been virtually free to use. Many broadcasters now fear the loss of their spectrum to more valuable telecom applications.

In this article we derive expressions for the value of both telecommunications and broadcast spectrum. An investigation of the capacity of the ether leads to the conclusion that, in the rare case of spectrum shortage, it is more reasonable to take spectrum from telecoms and give it to broadcasters than the other way around.

In most parts of the world, the radio spectrum is under-utilized. Large portions of the FM and UHF bands are unused in many countries, even in the developed world. In places where the demand for spectrum is higher, applications such as simulcast, loop programming and near-video-on-demand (NVoD) show that there is no acute shortage. There are, however, local problems in densely populated areas, along regional and country borders and at certain times of the day (e.g. during the rush hours) or the year (e.g. New Year).

Governments have responded to this apparent shortage by auctioning spectrum. The Federal Communications Commission in the United States took the lead in 1994, but the incredible sums raised by the European 3G auctions in 2000 and 2001 generated worldwide awareness and interest in the potential value of spectrum. Although economists generally agree that an auction is the right mechanism for dividing a scarce resource, it is unclear whether the buyers paid the right price for their spectrum. That is the question we will try to answer.

The capacity of the ether

Before trying to assess the value of spectrum, we will derive a very simplified rough estimate of the amount of useful signal the ether can carry – its so-called “channel capacity”. Although the electromagnetic spectrum is unlimited, its useful range is limited. On the low-frequency side, it is limited by the lack of useful bandwidth while on the high-frequency side, by propagation problems, as the penetration depth is more or less proportional to the wavelength.

Shannon [1] has shown that the channel capacity $C$ of any medium can be written as:

$$C = B \cdot \log_2 \left( 1 + \frac{S}{N} \right)$$

... where $C$ is the channel capacity (bit/s), $B$ is the bandwidth (Hz) and $S/N$ is the signal-to-noise power ratio.
In our simplified analysis, the noise is only caused by distant interfering transmitters on the same frequencies. In reality, many other noise sources exist, which we will not consider here.

In analogue operations, it is common practice to try to obtain a high S/N with a limited RF bandwidth – usually in the same order of magnitude as that of the analogue signal itself.

In the digital world, this constraint no longer exists: here, one can freely exchange bandwidth and signal-to-noise ratio to obtain the required channel capacity. A high signal-to-noise ratio can be achieved by keeping interfering transmitters at a large distance from the region being served, and limiting the power and antenna height of the interferers so as to hide them beyond the horizon. A typical frequency reuse factor (1/F) in analogue broadcasting is 1/15, which means that every cell (the area served by a single transmitter or by multiple transmitters synchronized on the same frequency) is surrounded by F – 1 (i.e. 14) more or less equal cells using other frequencies. As another extreme, we can use modulation techniques such as Code Division Multiplex (CDM) or Ultra Wide Band (UWB) and correlating receivers to obtain a useful channel capacity, even at S/N < 1. The question then arises: "Is there an optimal combination of B and S/N that maximizes the channel capacity?".

Consider a zone, divided in F equal cells, each containing a transmitter with power P. Every transmitter in the zone uses a band of width B/F centred around a different frequency, and the bands of the zone are not overlapping. The total bandwidth for serving the zone is B. The zones are repeated periodically, so that any plane can be covered in this manner, still with bandwidth B. Transmitters and receivers use antennas that are isotropic in the plane. This analysis therefore does not apply to point to point links.

The power density of a transmitter’s signal at a distance x can be approximated as

\[ P/4\pi x^m, \]

where m is an exponent that depends on the environment in which the waves propagate: in free space m = 2, whereas in areas with dense “clutter” due to buildings or vegetation, m can reach 4 or even more. Fig. 1 shows m as a function of transmitter height for three different frequencies, at a distance of 10 km from the transmitter and with a receiver antenna height of 10 m above ground level (agl). The curves are based on those in ITU-R Recommendation P.1546[2]. The upper line is m derived from the Okumura-Hata model with identical parameters, as described in Annex 7 of the same ITU Recommendation.

Let us assume there are n cells per unit surface, and that the cells can be approximated by a circle. This can of course never be the case in reality, because circles will either overlap each other or leave patches uncovered, but it gives us a reasonable approximation without the unwieldy formulas involved when using true geometric tessellations 1. The service area of every transmitter then stretches to a radius of \[ \sqrt{1/n}, \]

and the signal S at that distance from the wanted transmitter is given by the power density:

\[ S = P(\pi n)^{m/2}/4\pi. \]

All other transmitters operating at the same frequency are interferers. If we suppose their signals are not correlated, we can determine their interfering power at the receiver by adding their power densities. We will

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1. An arrangement of polygons without gaps and not overlapping, especially in a repeated pattern.
approximate the radius of a zone by $aF/\pi n$. The closest interfering transmitter is then approximately situated at a distance $a = \sqrt{F/\pi n} - \sqrt{1/\pi n}$.

We then obtain the total interfering power $N$ by integrating over the plane:

$$H = \int_{a}^{\infty} \frac{n}{F} \left( \frac{P}{4\pi a^{m}} 2\pi x dx \right) = \frac{nP}{2F} \left\{ \frac{a^{2-m}}{2-m} \right\} \quad \text{(for } m < 2)$$

... from which the signal-to-noise ratio is given by:

$$\frac{S}{N} = \frac{F(m-2)(2\sqrt{F-1})^{m-2}}{2}$$

We can substitute this expression in Shannon’s formula, where we should also replace $B$ by $B/F$, as $B$ is the bandwidth required for serving the entire zone, but every cell in the zone only uses $B/F$ Hz. Plotting $C/B$ (in bit/s/Hz or simply “bit”) as a function of $F$ (the inverse frequency reuse factor) and $m$, yields Fig. 2.

At this point a few observations can be made. They are all quite obvious to professional spectrum planners, but are worth stating for newcomers:

- The result is independent of the transmitter power $P$. This is due to our interference-limited approach. In practice the power of the transmitter should be chosen sufficiently high to overcome other noise sources such as natural and man-made noise and, probably even more importantly, receiver noise.

- The result is independent of the cell size ($1/n$). This means that any finite channel capacity (subject to the capacity of the given bandwidth and, ultimately, even the entire usable electromagnetic spectrum) can be obtained in any cell, however small. If these cells can be connected by a fixed (wired) network, in practice it means that the capacity for telecommunications is virtually unlimited. This is not the case for broadcasting, where large cells are generally preferred. As a consequence, in case of spectrum shortage, it makes sense to transfer spectrum from telecoms operations to broadcasting applications, and reduce the cell size for telecommunications as necessary.

- The capacity is zero for $m = 2$, i.e. free-space propagation. In practice this need not concern us, as we did not take into account the masking effect of the horizon. In that respect, our approximation can be considered a worst case. The curves of ITU-R Recommendation P.1546 show the masking effect very clearly, but it does not show up in Fig. 1 because of the short distance of only 10 km between the transmitter and receiver.

- There is indeed an optimum for $C/B$, for $F$ somewhere between 2 and 3. The optimum is hardly significant for most practical cases ($m < 4$), but some improvement in efficiency is possible with more fre-
quency reuse (smaller $F$) than is the case with current frequency-planning parameters (apart from CDM/ UWB planning). This will probably require new modulation schemes such as CDM or UWB.

A reasonable conservative value for $C/B$ in current planning appears to be 0.1 bit. Appendix A derives some real values for actual spectrum uses, that seem to support this finding – especially for the portable and mobile environment. This is not surprising as fixed applications, and notably satellite communications, use directional antennas instead of the isotropic ones our calculation was based upon. Fixed communications therefore achieve a higher spectral efficiency.

Note that this result is based on the assumption of an interconnecting fixed network. The case of full (isotropic) wireless interconnection has been studied elsewhere [3], and leads to the result that the channel capacity is proportional to the square root of the number of cells.

### A value for telecom spectrum

Economic theory has a range of techniques for valuing goods and services. Most of them are unsuited for determining the value of spectrum. A common approach for a public resource such as spectrum is to calculate the cost of recycling it. In the case of spectrum this amounts to the cost of spectrum planning and administration. Some countries have indeed valued their UMTS spectrum on this basis, but the values obtained are far lower than those obtained by auctioning the spectrum.

Another possible approach is to calculate the net present value of exploiting the spectrum over the licensing period. This method can only deliver a very speculative result, due to the uncertain nature of the 3G business models. Moreover it is unclear what percentage of this value should be returned to the community for the use of the spectrum, and how this should complement or replace normal income taxes for the operators involved.

The most common approach has been to auction the spectrum. Economists generally agree that this is the fairest way of valuation, if done properly [4][5]. Unfortunately, this part of economic theory does not help the bidding parties to determine their bid.

We will try to value spectrum by a technique called substitution: if the spectrum were not available, how could we deliver a similar service and at what price? There are of course no direct alternatives for 3G: fixed broadband Internet access has similar functionality but lacks the portability and mobility, while mobile telephony and broadcasting lack functionality. All these services give a lower bound to the value of 3G.

We will consider another substitution, inspired by the previous section. There we investigated the substitution of bandwidth by signal-to-noise ratio, which did not show a clear winner. We will now take a look at the substitution of bandwidth by cellularity. The previous section showed us that we can obtain virtually any channel capacity in a given bandwidth over any surface by increasing the cellularity. So there are several ways to serve a given area with a certain traffic demand: either with a low cellularity and a large bandwidth, or with a low bandwidth and a high cellularity. The licensing policy should promote the most cost-effective solution.

Suppose the area to be served is divided into $n$ equal cells. The traffic density (traffic per unit area) in the area is presumed to be constant, and the total traffic is $T$. If we denote the spectral efficiency as derived in the previous section as $e = C/B$, then the required bandwidth for serving the area is:

$$B = \frac{T}{ne}$$

(Equation 1)

The cost of equipping and operating a cell consists of a fixed part $F$, a variable part and the price of the spectrum or licence fee. The fixed part comprises the cost that is independent of the cell traffic, e.g. the operating and capital expenses of the fixed network interconnecting the base stations. The variable part consists of the costs proportional to the traffic, and can be expressed as $aT/n$, where $a$ is an appropriate proportionality factor. This includes the part of the cost of the base stations that is proportional to the traffic or the area covered, e.g. the mast carrying the antennas, the transmitter and its power consumption.
If we denote by $L$ the licence-fee-per-unit-bandwidth for the entire area, the “part per cell” can be written as:

$$\frac{LB}{n} = \frac{LT}{en^2}$$

The total cost $P$ for serving the area with $n$ equal cells then is:

$$P = n \left( F + \frac{aT}{n} + \frac{LT}{en^2} \right) = nF + aT + \frac{LT}{en}$$

By differentiating the above equation with respect to $n$ (i.e. $dP/dn$), we obtain the minimum cost when $dP/dn = 0$, or $F = LT/en^2$.

The optimum number of cells then becomes:

$$n = \sqrt{\frac{LT}{enF}}$$

(Equation 2)

From Equation 1 we can obtain the minimum number of cells $n_{min}$ required to fulfil the traffic need $T$ in a given bandwidth $B$:

$$n_{min} = \frac{T}{eB}$$

(Equation 3)

From Equations 2 and 3 we can then derive the smallest licence fee $L_{min}$ needed to obtain this number of cells:

$$L_{min} = \frac{TF}{eB^2}$$

... or, the licence fee for the entire band $B$:

$$L_{tot} = B \cdot L_{min} = \frac{TF}{eB} = n_{min} \cdot F$$

(Equation 4)

Form: 1 2 3

The second form of Equation 4 illustrates the economic law of demand ($T$) and supply ($B$). It also shows that our approach, in a certain sense, encompasses the net present value approach to spectrum valuation: in our findings, the licence fee is proportional to the traffic $T$ which also generates the income for the operator. Note that the traffic density is not necessarily proportional to the population density; we will return to this at the end of this section. Also note that the value of the licence is always positive.

The third form of Equation 4 shows that there is a direct relation between the licence fee and the number of cells in the network. This means that a government can, in principle, influence the degree of cellularity of the network by determining the (minimum) licence fee. The third form of the equation also indicates that the licence fee equals the fixed cost of the network which – in its limiting form, with extremely small cells and negligible hardware cost per cell – amounts to the cost of the fixed network underlying the wireless network. This is also reflected in the total cost of the network, which becomes:

$$P = 2nF + aT$$
This result shows that wireless mobile communications cost approximately twice as much as their fixed counterparts. It is supported by numerous citations in the press and on the Internet [6][7].

However plausible this result, it still does not provide an explanation for the wide range of prices paid for the European UMTS licences, even after normalizing for the population, the number of licences and the duration of the licence (see Fig. 3).

A possible explanation can be found in the networking effect, also known as Metcalfe’s law. It can easily be shown that network traffic, and thus the value of a network, increases as \( n(n-1) = n^2 \), where \( n \) is the number of network nodes or participants. If we recalculate Fig. 3 not per head of population but per head squared, we obtain values that are much closer spaced, as shown in Fig. 4.

If the prices were indeed based on the network effect, it is doubtful whether this was the right decision: from the beginning, 3G networks will certainly be connected to other networks such as the telephone network and the Internet. Furthermore, through roaming agreements, the networks will not be confined to a single country.

**Estimating the market share**

In the previous section we rated the value of spectrum for telecom usage or “unicast”. In the next section we will derive a value for broadcast spectrum, but first we need a model to estimate market shares for broadcast stations.

Let us first try an intuitive approach. Suppose all stations address the same market, and offer similar programming. “Similar” means programming that attracts an audience with the same probability \( p \). Most people will tune to their favourite station, and stay tuned with probability \( p \). A fraction \((1-p)\) however will not be satisfied with the current programming, and will tune to another station. Due to the similarity of programmes, again a fraction \( p \) will stay there, but \((1-p)^2\) will still not be content with the programming of station 2, and will move on to station 3. Continuing in this manner, the market shares of the different stations will be \( p, (1-p)p, (1-p)^2p, (1-p)^3p, ... \) the well known geometric distribution. Fig. 5 compares this result with some actual European markets, and shows a satisfactory correlation.
This model has a few drawbacks: first, the geometric distribution never reaches zero, so the model implies an infinite number of market participants. Second, we need the share of the market leader $p$ as a given parameter. In the following section we will need a model that estimates the market shares for the smallest player, given a finite number of participants ($N$).

We will therefore introduce an even more general model. Suppose we divide a unity length (100% market) in $N$ random pieces by randomly placing $N-1$ points on it. We subsequently order the pieces from small to large. We repeat the experiment a large number of times, and make a histogram of the different pieces. The result (for $N = 4$) is illustrated in Fig. 6.

It can easily be shown that the probability density function of the smallest part, normalized for unity surface under the curve, is given by:

$$E(x) = N(N-1)(1-Nx)^{N-2}$$

... (where $0 \leq x \leq 1/N$), from which it is equally easy to deduce the average value of the smallest “market participant” as $1/N^2$.

The interested reader is encouraged to pursue this “experiment” a bit further, even if not relevant to spectrum valuation. He/she will discover that the most likely value of the smallest participant is always zero, except for the trivial cases $N = 1$ (monopoly) and $N = 2$ (duopoly). All other market situations are inherently unstable, as the smallest player tends to disappear. Another interesting property is that newcomers to a market tend to increase (the average value of) the market share of other (existing) small players, always mainly at the expense of the market leader.

**A value for broadcast spectrum**

A plausible aim for a government in attributing broadcast spectrum could be...
to maximize the choice for the consumers, by providing the maximum number of broadcasters that both the spectrum and the market allow. In a pure commercial situation, the broadcaster’s income originates solely from advertising, which in turn is determined by the total advertising spending in that medium and the market share of the broadcaster in the region under consideration.

Suppose the broadcast spectrum allows for \( N \) broadcasters in the region, in which an amount \( A \) is spent for advertising in the broadcast medium under consideration. Suppose every broadcaster pays a spectrum licence fee \( L \), and the station with the smallest market share \( 1/N^2 \) (from the previous section) has a production cost \( P \) (including rights and distribution costs), then in order for this station to survive, the licence fee cannot be higher than:

\[
L < \frac{A}{N^2} - P
\]

... which turns out to be negative in many cases, especially in small countries or areas. A negative licence fee means that the government should subsidize the stations instead of charging a licence fee, and this is precisely what happens with Public Service Broadcasters in many countries.

**Conclusions**

We have shown that, given an appropriate modulation scheme and frequency reuse, there need never be a real shortage of spectrum for telecommunications applications. This is not the case for broadcasting. It therefore seems reasonable, in case of spectrum shortage, to transfer spectrum allocations from telecommunications to broadcasting.

It may seem paradoxical to plead for a spectrum transfer from telecoms to broadcasting, while at the same time showing that telecom spectrum always has a positive value, and broadcast spectrum in many cases has a negative value. The plea only holds for regions where there is spectrum shortage, and thus both types of spectrum have a positive value. In most regions of the world, there is no spectrum shortage, and it may be more appropriate to transfer spectrum from broadcasting to telecoms, to alleviate the costs of telecom operations by allowing larger cell sizes.

A corollary of the strictly positive value of telecom spectrum is that the “free” or unlicensed bands such as ISM (2.4 GHz) and UNII (5 GHz) will continue to attract more traffic, and may soon become overcrowded and unusable.
Bibliography


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[6] “1/2 the cost of UMTS will go to the Governments!”
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http://www.iis.ee.ic.ac.uk/mtn/lectures/129046_Presentation.pdf

[7] “The early European auctions conducted in 2000 raised nearly $100 billion. An additional expense of at least $100 billion will be required to build the infrastructure necessary to provide service”
P. Cramton, “Spectrum Auctions”

[8] K. Appel and W. Haken: Every planar map is four colorable. Part I – Discharging


Appendix A:
Some actual values of ether capacity

Case 1: Digital geostationary satellites (Ku band)

A typical transponder of 33 MHz carries a net load of some 30 to 40 Mbit/s. This looks like a spectral efficiency of 1 bit/s/Hz. Taking into account polarization diversity, through which every frequency can be used twice, we obtain 2 bit. With orbital positions 9° apart, there are 20 positions theoretically possible, all of which can send the same frequency to a single region. However we should apply frequency reuse (in the order of 1/4) to be able to cover the whole of the earth’s surface with the same amount of signals. This then reduces the spectral efficiency at the equator to:

\[ 2 \text{bit/s/Hz} \times 20 \text{ positions} / 4 = 10 \text{bit/s/Hz}. \]

This efficiency decreases with increasing latitude, and vanishes near the poles where geostationary satellites are invisible.
Of all the systems considered, geostationary satellites have by far the highest efficiency, especially at the equator. They can be considered the ideal broadcasting medium for fixed reception.

**Case 2: analogue TV broadcasts in bands III, IV and V**

The video bandwidth is 5 MHz; let us assume the $S/N$ at the edge of the coverage area is 20 dB, which gives a net bitrate (using Shannon’s formula) of approximately 33 Mbit/s. The RF bandwidth is 5.75 MHz (video only, 0.75 MHz lower sideband), so the rate per unit bandwidth is:

$$\frac{33}{5.75} = 5.74 \text{ bit/s/Hz}.$$ 

However we must take into account that full coverage is achieved with multiple frequencies. In Europe, the planning of the 60 channels available was done on a basis of four networks per country, so that the frequency reuse factor for a single network is approximately $4/60 = 1/15$.

The net rate per unit bandwidth becomes:

$$\frac{5.74}{15} = 0.38 \text{ bit/s/Hz}$$

Apparently, analogue television transmission has a very good performance. This is due to the fact that in analogue TV, the transmitters are located far beyond each other’s horizon, as illustrated by the small frequency reuse factor. Some further increase is possible by polarization diversity. Note that, in general, only fixed reception (with a rooftop antenna) is possible.

**Case 3: digital terrestrial television (DVB-T)**

If digital television simply replaces analogue TV, then we only have to substitute the new bitrate in the calculation of case 2: instead of 33 Mbit/s we then have 22 Mbit/s for fixed reception, down to 12 Mbit/s for portable or mobile reception.

However the situation can be enhanced nearly fourfold by introducing single-frequency networks (SFNs). In the optimal case, the frequency reuse factor can be increased to 1/4, compared to 1/15 in current analogue planning. “Optimal case” here means a set of single-frequency networks that are neither too large (to avoid self-interference) nor too small. 1/4 is based on the results obtained by Appel and Haken, who have shown [8][9] in 1976 that any map can be covered with four non-adjacent colours.

**Case 4: analogue FM radio broadcasts in band II**

Assuming an audio bandwidth of 16 kHz and an $S/N$ of 42 dB (mono) at the edge of the coverage area, we obtain a net bitrate of 224 kbit/s. The FM band (87.5 - 108 MHz) typically has room for 9 stations, which results in:

$$9 \times 224 / (108 – 87.5) = 98 \text{ bit/s/kHz}$$

... or some 0.1 bit/s/Hz.

**Case 5: digital radio (DAB)**

This robust system transmits an information rate of 1.5 Mbit/s in a channel of approximately 1.5 MHz, for an efficiency of 1 bit/s/Hz. A frequency reuse factor of 1/9 (a little larger than FM radio) leads to a net efficiency of 0.11 bit. Single-frequency networks can increase this value about twofold.
Case 6: GSM

The downstream frequency band is 35 MHz wide (925 - 960 MHz), and carries 124 channels of eight conversations (at approximately 13 kbit/s), for a total of 12.9 Mbit/s at full load. This results in 0.368 bit/s/Hz. Taking into account the typical frequency reuse factor of 1/7, we obtain 0.053 bit/s/Hz. This fits much better with the result found in the first section of this article, than for analogue broadcasting, mainly because GSM transmitter towers operate more or less within each other’s line of sight.

Summary

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