Simplified procedure for evaluating the coverage area of a multifeed shaped antenna
– Application to the planning of a broadcasting satellite system

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Shaped antenna beams offer near-constant antenna gain throughout the nominal coverage area of a geostationary satellite. Also, they provide a fast roll-off in antenna gain outside the footprint, thus facilitating the solution of interference problems.

In this article, the authors describe a simplified analytical algorithm which can produce a set of shaped antenna PFD contours on the Earth's surface. These contours can be used as a coverage mask by the antenna manufacturer and can also provide the data input to existing software programs that calculate out-of-beam interference levels.

1. Introduction

Shaped antenna beams have been used for many years by direct-to-home (DTH) satellites, operating in the 10.7 – 11.7 GHz band. It seems very likely [1] that they will also be introduced in the 11.7 – 12.5 GHz satellite broadcasting band, when it is replanned during the forthcoming revision of the WARC-77 Plan (which was based on the use of elliptical antenna beams).

The main advantages of using shaped antenna beams are:

– near-constant antenna gain inside the footprint;

Original language: English
Manuscript received 10/12/95.

– fast roll-off characteristics outside the footprint, thus facilitating the solution of interference problems (which can be a very important political consideration in the case of satellites which operate with supranational beams).

Nevertheless, it should be noted that shaped antenna beams have two main disadvantages when compared with elliptical antennas (whose unavoidable out-of-coverage radiation is generally well defined). First of all, it is not generally possible to predict the power flux density (PFD) contours of a shaped antenna beam on the Earth's surface, prior to the construction of a real satellite antenna [1]. Secondly, due to the complexity of shaped antennas, the CAD software tools that are currently available for designing them cannot easily provide a data input to computer programs which are used to calculate the interference levels.

One possible approach to the planning of shaped antenna satellite systems is based on the use of realistic PFD contours for each service area (as previously agreed among the administrations concerned). Each agreed set of contours for a given service area can then form the basis of the coverage mask (i.e. antenna radiation pattern) to be respected by the antenna manufacturer, and may also be utilized when performing the interference calculations[1]. In this approach, it is of fundamental importance that suitable methods are available for producing realistic PFD contours on the Earth's surface. CCIR Report 558-4 [2] describes some procedures that are suitable for this task but, as they are essentially graphical, some difficulties may be encountered when they are being managed by a computer system.

In this article, a simplified analytical algorithm is described which can produce, for a given service area, a set of shaped antenna PFD contours on the Earth's surface, suitable to be used as antenna masks in the planning procedure. The algorithm is also able to synthesize – as a first step – an approximate design for a multifeed shaped antenna and, at the same time, it can give an indication of the antenna's physical complexity. Furthermore, as the algorithm provides a suitable data input to interference-calculating computer software, it can be treated easily by this software. At the RAI Technical Headquarters, an interference-comput-

1. It should be noted that shaped antenna beams can still be introduced within a planning approach that is based on elliptical antenna beams. In this case, it is sufficient that the PFD contours produced by the shaped antenna beam on the Earth's surface should be compliant with the radiation pattern of the relevant elliptical antenna. However, the major roll-off advantages of the shaped antennas are lost during the planning calculations.

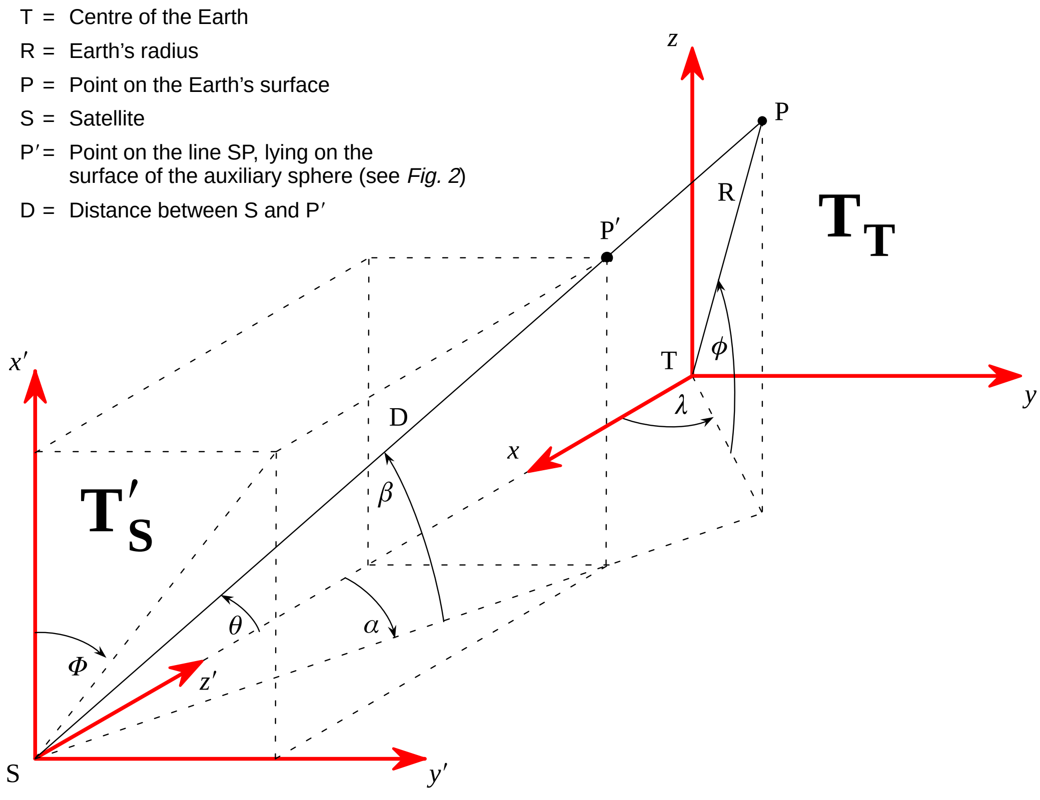


Figure 1

ing program has been run successfully, making use of elliptical and/or shaped antenna beams.

Within the limits imposed by the adopted simplifications (see *Section 6.*), the PFD contours that have been computed for assigned configurations of feeds have shown a satisfactory match with the results obtained using exact design procedures (e.g. TICRA software), as shown in *Section 6*.

2. The reference system

We assume the use of two Cartesian reference systems $\mathbf{T_T}(x,y,z)$ and $\mathbf{T'_S}\,(x',y',z')$ to define the location of points in space (see *Fig. 1)*.

The reference system $\mathbf{T_T}(x,y,z)$ has its origin at the centre of the Earth (T). The z axis is coincident with the south-north direction, oriented towards the north; the x axis coincides with the direction of the satellite (S), relative to T.

The reference system $\mathbf{T'_S}\,(x',y',z')$ has its origin at the satellite (S). The x' and y' axes are parallel (both in direction and orientation) with the z and y axes of $\mathbf{T_T}$ respectively; the z' axis of $\mathbf{T'_S}$ is coincident with the direction of the x axis of $\mathbf{T_T}$ but is of opposite orientation.

Let us define in $\mathbf{T_T}$ a polar reference system $[\lambda,\phi]$, where λ is the longitude and ϕ is the latitude of a point (P) on the surface of the earth (see *Fig. 1*).

We note that the longitude (λ) of point P is referred to the line joining the centre of the Earth (T) with the satellite (S) and is regarded as positive in the direction of east. The latitude (ϕ) of point P is referred to the equator and is regarded as positive in the direction of north.

Let us define in $\mathbf{T'_S}$ two polar reference systems $[\alpha,\beta]$ and $[\theta,\Phi]$ as indicated in *Fig. 1*. Along with λ and ϕ, the positive angular directions of α, β, θ and Φ are also indicated in *Fig. 1*.

From among the defined reference systems, we will use the most suitable one at each juncture. Also, for reasons of simplicity, we will treat the Earth as perfectly spherical; this is well justified by the radioelectric approximations that will be introduced in *Sections 4* and *5*.

Let:

R = Earth's radius (6378 km approx.)

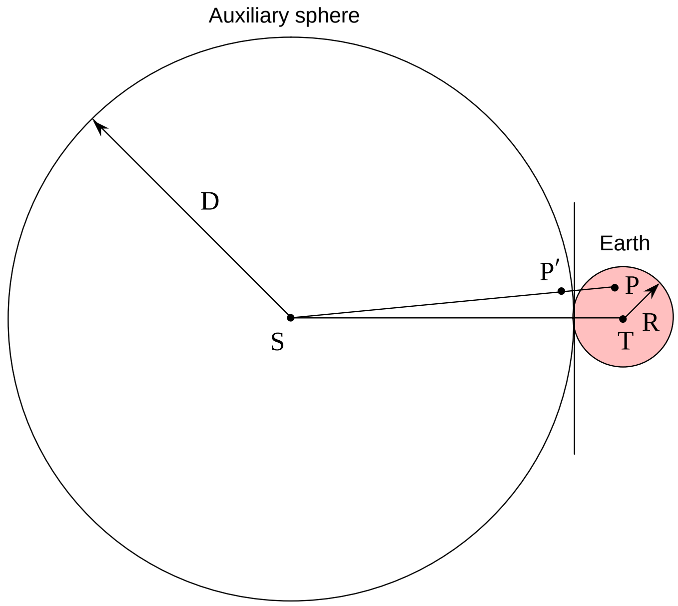


Figure 2

P(λ,ϕ) = a generic point on the Earth's surface, defined by its geographical coordinates λ (longitude) and ϕ (latitude)

D = Distance of the satellite from the Earth's surface (35,860 km approx.), which is also equal to the distance $\overline{SP'}$

In the reference system $\mathbf{T_T}$, the Cartesian coordinates of P are:

$$\begin{aligned} x &= \mathrm{R}\cos\phi\cos\lambda \\ y &= \mathrm{R}\cos\phi\sin\lambda \\ z &= \mathrm{R}\sin\phi \end{aligned} \qquad (1)$$

Let us consider a sphere of radius D, with its centre at the satellite (S). This sphere – here referred to as the *auxiliary sphere* – touches the Earth (assumed to be spherical) at a point on the line $\overline{ST}$ *(Fig. 2)*. For each point P on the Earth's surface, there is a corresponding point P′(x',y',z') on the auxiliary sphere, at the intersection of the auxiliary sphere and the line $\overline{SP}$.

Referring again to *Fig. 1*, the Cartesian coordinates of P′ in the reference system $\mathbf{T'_S}$ are:

$$\begin{aligned} x' &= \mathrm{D}\sin\theta\cos\phi = \mathrm{D}\sin\beta \\ y' &= \mathrm{D}\sin\theta\sin\phi = \mathrm{D}\cos\beta\sin\alpha \\ z' &= \mathrm{D}\cos\theta = \mathrm{D}\cos\beta\cos\alpha \end{aligned} \qquad (2)$$

The point P′(α,β) can be thought of as a "mapping" of P(λ,ϕ) onto the auxiliary sphere: the point P′(α,β) thus represents the point P(λ,ϕ) as seen from the satellite (S).

If the geographical coordinates λ and ϕ of P(λ,ϕ) are known, we can compute the Cartesian coordinates x, y and z of $\mathrm{P}(x,y,z)$ in $\mathbf{T}_\mathrm{T}$, by means of *Equations 1*. We can then find the α and β coordinates of $\mathrm{P}'(\alpha,\beta)$ using *Equations 3*:

$$\alpha = \arcsin\left(\frac{y}{\sqrt{(\mathrm{D}+\mathrm{R}-x)^2+y^2}}\right)$$
$$\beta = \arctan\left(\frac{z}{\sqrt{(\mathrm{D}+\mathrm{R}-x)^2+y^2}}\right) \qquad (3)$$

Summarizing, α and β are the directional coordinates in the reference system $\mathbf{T}'_\mathrm{S}$ of point P' and, hence, of point P (on the Earth's surface) as "seen" from the satellite (S).

3. The geometry of the problem

Let us imagine a service area on the Earth's surface which has the shape of a polygon whose vertices are at the following points:

$$\mathrm{P}_1(\lambda_1,\phi_1)\ ,\ \mathrm{P}_2(\lambda_2,\phi_2)\ \ldots\ \mathrm{P}_\mathrm{M}(\lambda_\mathrm{M},\phi_\mathrm{M}) \qquad (4)$$

By means of *Equations 1* and *3*, we can calculate in $\mathbf{T}'_\mathrm{S}$ the directional coordinates α and β of the polygon vertices. The polar coordinates of these vertices, as seen from the satellite, are thus:

$$\mathrm{P}'_1(\alpha_1,\beta_1)\ ,\ \mathrm{P}'_2(\alpha_2,\beta_2)\ \ldots\ \mathrm{P}'_\mathrm{M}(\alpha_\mathrm{M},\beta_\mathrm{M}) \qquad (5)$$

Let $\mathrm{P}_0(\lambda_0,\phi_0)$ be a point on the Earth's surface, located at the "centre" of the polygonal service area. Furthermore, let α_0 and β_0 be the directional coordinates in $\mathbf{T}'_\mathrm{S}$ of this point, as seen from the satellite.

The problem lies in designing an antenna to be placed at the satellite (S) such that its "–x dB" isoflux contour on the Earth's surface completely contains the service area defined by the vertices given in *Equation 4*. However, a well-designed antenna should satisfy not only the preceding condition but also the constraint that the roll-off outside the "–x dB" isoflux contour should be sufficiently rapid to minimize any potential out-of-beam interference problems.

4. Approximate analysis of the geometry of the parabolic antenna

Let us assume that the satellite antenna is an offset parabolic reflector. The origin of the reference system $\mathbf{T}'_\mathrm{S}$ is assumed to be at the focus of the parabola.

In order to simplify the calculations, we will refer the antenna geometry to a reference system $\mathbf{T}''_\mathrm{S}\,(x'',y'',z'')$, which is obtained by rotating

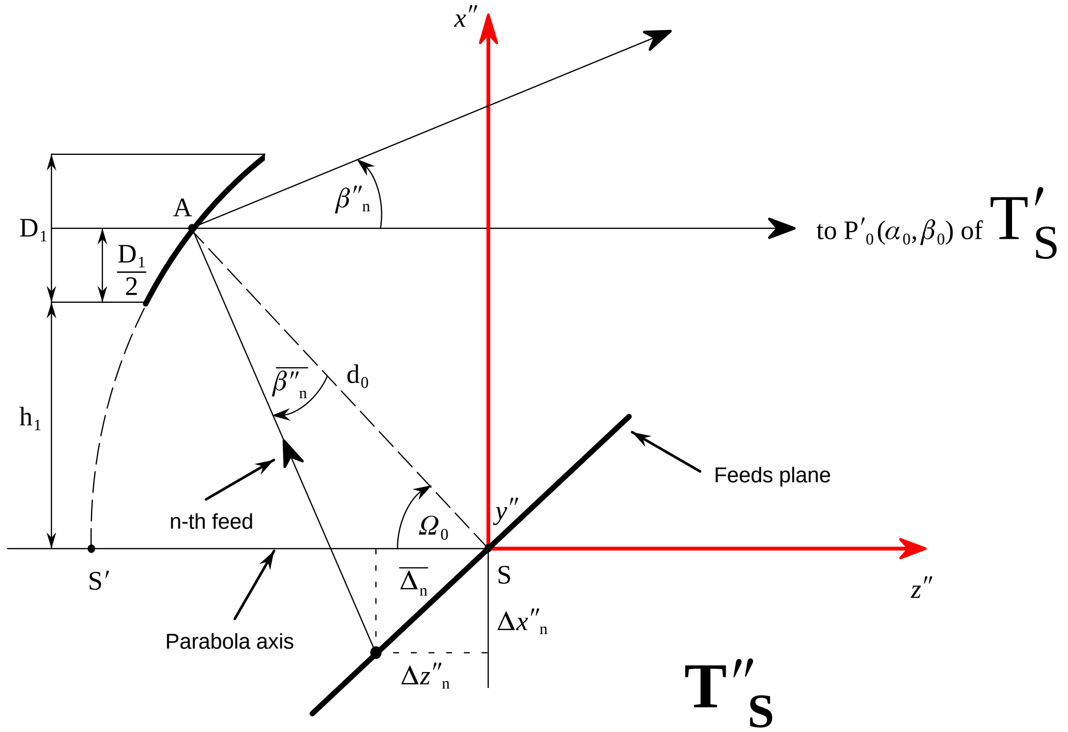


Figure 3

$\mathbf{T}'_S\,(x', y', z')$ around the origin S (which is kept fixed) and orientating the z'' axis of $\mathbf{T}''_S$ towards the point $P'_0(\alpha_0, \beta_0)$ in $\mathbf{T}'_S$ (which represents the centre of the service area).

In $\mathbf{T}''_S$ we define two further polar reference systems $[\alpha'', \beta'']$ and $[\theta'', \Phi'']$ which are related to the systems $[\alpha, \beta]$ and $[\theta, \Phi]$ of $\mathbf{T}'_S$ by:

$$\alpha'' = \alpha - \alpha_0 \qquad \theta'' = \theta - \theta_0$$
$$\beta'' = \beta - \beta_0 \qquad \Phi'' = \Phi - \Phi_0 \qquad (6)$$

Fig. 3 shows in $\mathbf{T}''_S$ the geometry of the parabola, intersected by the plane (x'', z'') of $\mathbf{T}''_S$. The positive sense of each angle is indicated on the figure.

Let:

- f = $|\overline{SS'}|$ (the focal distance of the parabola)
- Ω_0 = offset angle (the angle between the $-z''$ axis and the $\overline{SA}$ direction)
- h_1 = distance from the parabola axis to the lowest point of the reflector
- D_1 = reflector length measured along the x'' axis of $\mathbf{T}''_S$.
- d_0 = distance between the parabola focus point S and the point A (the conventional centre of the reflector as seen from S).

Due to the parabola geometry we have:

$$d_0 = \frac{2f}{1 + \cos\Omega_0}$$

Point A is identified in $\mathbf{T}''_S$ by its coordinates:

$$x''_A = h_1 + \frac{D_1}{2}$$

$$y''_A = 0$$

$$z''_A = -\,d_0 \cos\Omega_0$$

The feeds which illuminate the parabola surface are placed on a plane which passes through point S and is perpendicular to the $\overline{SA}$ direction.

Let us call $\Delta x''_n$, $\Delta y''_n$ and $\Delta z''_n$ the coordinates in $\mathbf{T}''_S$ of the n-th feed. We assume that this feed is directed towards A (obviously if the feed were placed at the parabola focus S we would have $\Delta x''_n = \Delta y''_n = \Delta z''_n = 0$).

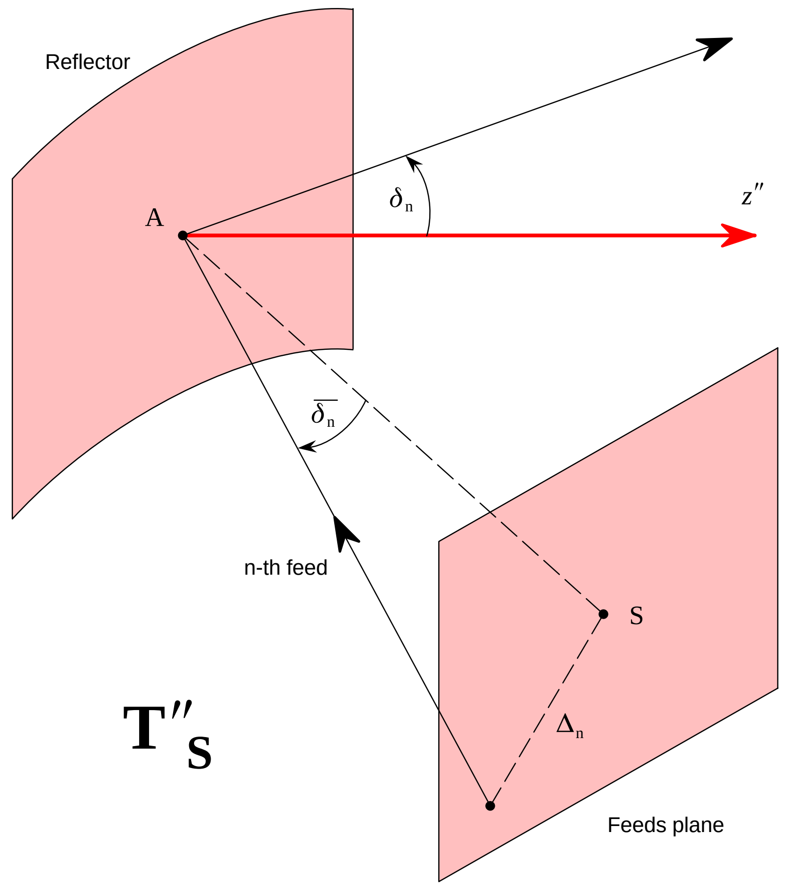


Figure 4

Let us consider the beam radiated by the n-th feed towards the point A of the parabola surface.

With reference to *Fig. 4*, let us denote by $\bar{\delta}_n$ the angle formed between the main direction of this radiated beam and the $\overline{SA}$ line.

This beam is reflected towards the Earth at an angle δ_n with respect to the z'' axis. The relationship between $\bar{\delta}_n$ and δ_n can be expressed by:

$$\frac{\tan\delta_n}{\tan\bar{\delta}_n} = \mathrm{BDF} \qquad (7)$$

where BDF (the beam deviation factor) can be approximated by the following relationship [3]:

$$\mathrm{BDF} = \left[1 - 0.72\, e^{-3.2\frac{f}{\tau D_1}}\right]$$

and where:

$$\tau = \frac{\cos\Omega_0 + \cos(\Omega_0 - \Omega_1)}{1 + \cos(\Omega_0 - \Omega_1)}$$

This expression for BDF has proved itself to be sufficiently accurate, as shown by the results presented in *Section 6*.

We can divide angle $\overline{\delta_n}$ into the two angles $\overline{\alpha''_n}$ and $\overline{\beta''_n}$ according to the following expression:

$$(\tan\overline{\delta_n})^2 = (\tan\overline{\alpha''_n})^2 + (\tan\overline{\beta''_n})^2$$

The angle $\overline{\alpha''_n}$ is measured in a plane passing through the line $\overline{SA}$ and the y'' axis, and $\overline{\beta''_n}$ is measured in the (x'', z'') plane (see *Fig. 3*).

Let the distance of the n-th feed from S be called Δ_n where:

$$\Delta_n = \sqrt{(\Delta x''_n)^2 + (\Delta y''_n)^2 + (\Delta z''_n)^2}$$

If we make:

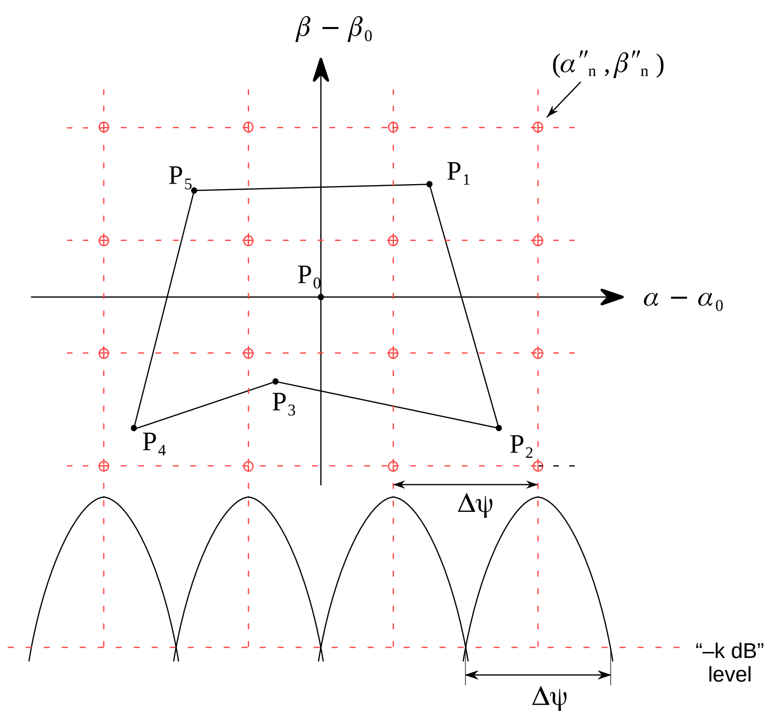
$$\overline{\Delta_n} = \sqrt{(\Delta x''_n)^2 + (\Delta z''_n)^2} = |\, d_0 \tan\overline{\beta''_n}\,|$$

It follows that:

$$\begin{aligned} \Delta x''_n &= -\overline{\Delta_n}\cos\Omega_0 \\ \Delta y''_n &= d_0 \tan\overline{\alpha''_n} \\ \Delta z''_n &= -\overline{\Delta_n}\sin\Omega_0 \end{aligned} \qquad (8)$$

The preceding formulas allow us (in a first approach) to position the feeds in the plane passing through S (the parabola focus) and normal to the $\overline{SA}$ direction.

Figure 5



With reference to *Fig. 5*, let us call $\Delta\psi$ the "–k dB" total aperture angle of the radiation diagram (in the Earth's direction) of the parabola fed by a single feed placed at the focus point. We assume that $\Delta\psi$ does not change significantly if the feed is placed outside the parabola focus, but still remains in the plane passing through S and normal to the $\overline{SA}$ direction (we recall that this feed is directed towards point A on the parabolic reflector).

We proceed as follows:

1. Let us assume that the geographical coordinates *(Equation 4)* of the service area vertices are known.
2. By means of *Equations 3*, we calculate in $\mathbf{T}'_S$ the directional coordinates *(Equation 5)* of the service area vertices.
3. By means of *Equation 6*, we represent in the polar reference system $[\alpha'', \beta'']$ of $\mathbf{T}''_S$ the directional coordinates of the service area vertices. For the i-th vertex $P'_i(\alpha_i, \beta_i)$, we have:

$$\alpha''_i = \alpha_i - \alpha_0$$
$$\beta''_i = \beta_i - \beta_0$$

4. Starting from the polar reference system $[\alpha'', \beta'']$ of $\mathbf{T}''_S$ we define by means of *Equations 2* a plane (u,v) related to the polar reference system $[\alpha, \beta]$ of $\mathbf{T}'_S$ by the following expression[2]:

$$u = \sin(\beta - \beta_0)$$
$$v = \cos(\beta - \beta_0)\,\sin(\alpha - \alpha_0)$$

In the (u,v) plane, we define a meshed network whose squares have dimension $\Delta\psi$ (the network axes are not necessarily parallel to the u and v axes). We assume that the service area is totally covered by this network.

The intersection point of the lines defining the network are assumed to be the directional coordinates α''_n, β''_n in $\mathbf{T}''_S$ of the beam reflected towards the Earth by the parabola surface, and coming from the direction of maximum radiation of the n-th feed (see *Fig. 5*).

5. By means of *Equation 7* we then correct the α''_n and β''_n angles in order to obtain the following angles:

2. If $\beta - \beta_0$ and $\alpha - \alpha_0$ are sufficiently small, we have:
$u \cong \beta - \beta_0$
$v \cong \alpha - \alpha_0$

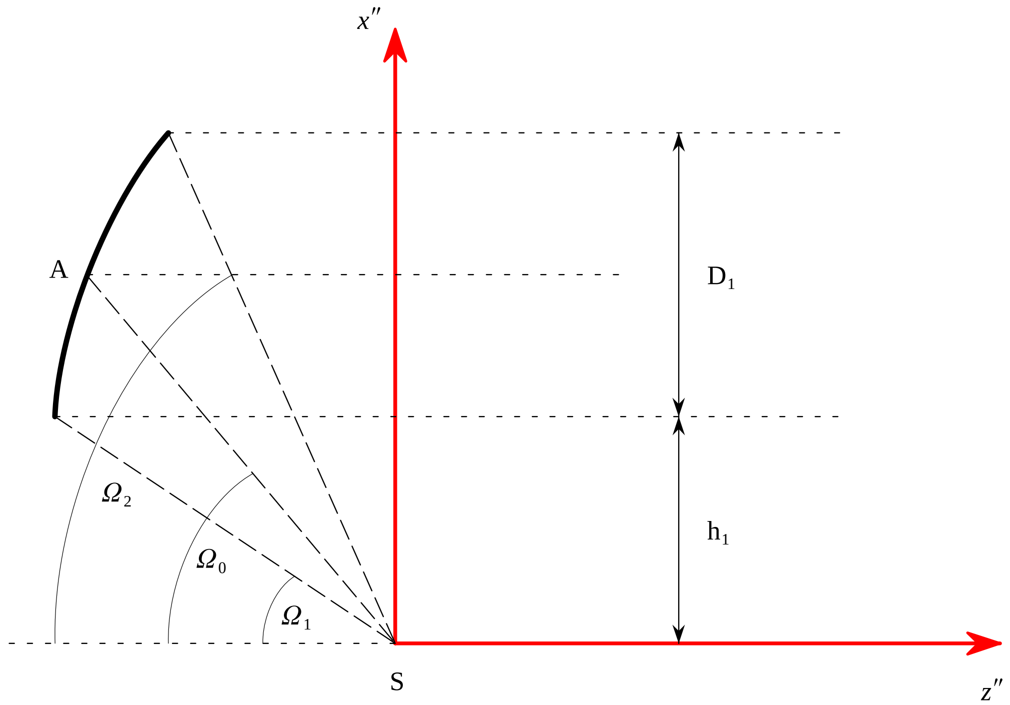


Figure 6

$$\tan \overline{\alpha''_n} = \frac{\tan \alpha''_n}{\mathrm{BDF}}$$

$$\tan \overline{\beta''_n} = \frac{\tan \beta''_n}{\mathrm{BDF}}$$

6. After fixing the geometrical characteristics of the antenna (the offset angle Ω_0, the Ω_1 and Ω_2 angles, the D_1/f ratio, the height h_1 and the diameter D_1 (see *Fig. 6*)) and knowing the directional coordinates α''_n , β''_n (towards the Earth) of the beam originating in the n-th feed, we calculate in $\mathbf{T}''_S$ (by means of *Equations 8*) the coordinates of the feeds to be positioned in the plane which passes through S and which is perpendicular to the $\overline{\mathrm{SA}}$ direction.

5. *Derivation of the radiation diagram of the shaped antenna*

Let us for the moment choose the reference system $\mathbf{T}'_S$ and the polar reference system $[\theta, \Phi]$ defined in $\mathbf{T}'_S$ (*Fig. 1*).

Suppose that (in the polar reference system $[\alpha, \beta]$ of $\mathbf{T}'_S$) we know the directional coordinates α_n, β_n of the reflected beam which points towards the Earth as a result of the n-th feed. By means of *Equations 2*, we can calculate the corresponding coordinates θ_n, Φ_n in the polar reference system $[\theta, \Phi]$ of $\mathbf{T}'_S$.

We will compute the antenna radiation diagram by introducing a few simplifying hypotheses. In the polar reference system $[\theta, \Phi]$ of $\mathbf{T}'_S$, let:

$$\underline{R}_n = \underline{x}' \sin\theta_n \cos\Phi_n + \underline{y}' \sin\theta_n \sin\Phi_n + \underline{z}' \cos\theta_n$$

where $\underline{R}_n$ is the unit vector in the direction of maximum radiation towards the Earth, resulting from the n-th feed. In order to compute the electric field produced by this radiation (see *Fig. 7*), we will consider a circular aperture of radius $d_n/2$ (the antenna output) which is obtained by projecting the antenna circular aperture of radius $D_1/2$ onto a plane which is perpendicular to the unit vector $\underline{R}_n$. By means of simple geometrical considerations, it is easy to derive from *Fig. 7* the diameter d_n:

$$d_n = D_1 \frac{\cos\left(\frac{\Omega_0}{2} + \delta_n\right)}{\cos\left(\frac{\Omega_0}{2}\right)}$$

Let $[r, \gamma]$ be a planar polar reference system defined in the plane of the aforementioned circular aperture (*Fig. 8* and *Appendix II*). Let the function $f_n(r, \gamma)$ represent the illumination of the aperture, caused by the radiation coming from the n-th feed.

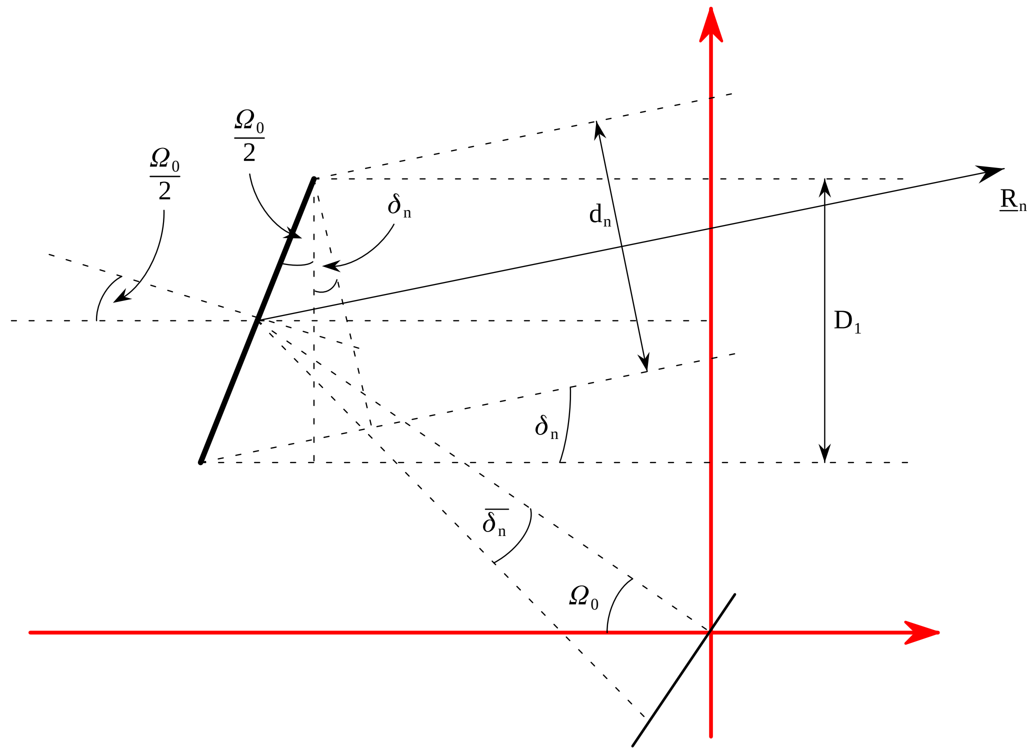


Figure 7

Let us assume that the n-th feed is directed towards point A on the parabolic reflector. Let us further assume that all the feeds have the same radiation function and that they all radiate in such a way that their resultant electric fields produced at point A are all in phase[3]. Let us also assume that $f_n(r,\gamma)$ has circular symmetry according to the law:

$$f_n(r,\gamma) = f_n(r) = 1-\rho+\rho\left[1-\left(\frac{2r}{\mathrm{d}_n}\right)^2\right]^m \qquad (9)$$

The direction of maximum radiation from the aperture is identified by the unit vector $\underline{\mathrm{R}}_n$.

The values to be assigned to m and ϱ depend on the aperture edge tapering. In *Appendix 1* we will show how to compute the values for m and ρ, starting from a knowledge of the (assigned) radiation pattern $g(\Omega)$ of the n-th feed.

The quantity ρ indicates the reduction of the electric field value at the edge of the circular aperture ($r = \mathrm{d}_n/2$) with respect to the electric field value in the centre of the aperture ($r = 0$).

In the polar reference system $[\theta, \Phi]$ of $\mathbf{T}'_S$, let:

$$\underline{\mathrm{R}} = \underline{x}'\sin\theta\cos\Phi + \underline{y}'\sin\theta\sin\Phi + \underline{z}'\cos\theta$$

where $\underline{\mathrm{R}}$ is unit vector of the generic direction θ, Φ in which we want to calculate the total electric field.

Let θ'_n be the angle between the unit vectors $\underline{\mathrm{R}}$ and $\underline{\mathrm{R}}_n$. As shown in *Appendix 2*, the electric field $\mathrm{E}_n(\theta, \Phi)$ – radiated in the direction $\underline{\mathrm{R}}$ from a plane circular aperture having radius $\mathrm{d}_n/2$, perpendicular to $\underline{\mathrm{R}}_n$ (*Fig. 7*) and illuminated by the function given in *Equation 9* – is a function of the angle θ'_n only:

$$\mathrm{E}_n(\theta, \Phi) = \mathrm{E}_n(\theta'_n)$$

From this (see *Appendix 2*):

$$\mathrm{E}_n(\theta'_n) = \pi\left(\frac{\mathrm{d}_n}{2}\right)^2 \times \left[2(1-\rho)\,\frac{\mathrm{J}_1(x)}{x} + \rho\,\frac{2^m m!\,\mathrm{J}_{m+1}(x)}{x^{m+1}}\right]$$

3. This is not a limitation of the theory. It is sufficient to add individual delays to each feed, in order to compensate for the different distances between each feed point on the feeds plane and point A on the parabolic reflector.

where:

$$x = d_n \frac{\pi}{\lambda} \sin\theta'_n$$

We can calculate θ'_n as follows (recalling that it is the angle between the unit vectors $\underline{R}$ and $\underline{R}_n$):

$$\underline{R}_n \cdot \underline{R} = \cos\theta'_n$$

Consequently, we have:

$$\theta'_n = \arccos[\sin\theta \sin\theta_n(\cos\Phi \cos\Phi_n + \sin\Phi \sin\Phi_n) + \cos\theta \cos\theta_n]$$

Let α_n be the fraction of the power radiated by the n-th feed, having assumed that the total power radiated by the N feeds is equal to unity. Expressing this mathematically:

$$\sum_{n=1}^{N} \alpha_n = 1$$

The resultant electric field $E(\theta, \Phi)$ is given by the vectorial sum of the fields radiated by the N feeds in the direction $\underline{R}$.

As already mentioned, we assume that the individual feeds have been suitably delayed to produce in-phase electric fields at point A on the parabolic reflector (all the feeds point towards A).

The resultant electric field in the θ, Φ direction is given by:

$$E(\theta, \Phi) = \sum_{n=1}^{N} \sqrt{\alpha_n}\, E_n(\theta'_n)$$

The corresponding power radiated in the θ, Φ direction is given by:

$$W(\theta, \Phi) = \left| \sum_{n=1}^{N} \sqrt{\alpha_n}\, E_n(\theta'_n) \right|^2$$

Let us refer the power $W(\theta, \Phi)$ (radiated in the θ, Φ direction) to the power radiated in the direction of a central point within the service area, denoted by $P_*(\theta_*, \Phi_*)$. The normalized power we obtain is given by:

$$\overline{W}(\theta, \Phi) = \frac{\left| \sum_{n=1}^{N} \sqrt{\alpha_n}\, E_n(\theta'_n) \right|^2}{E^2(\theta_*, \Phi_*)} \qquad (10)$$

The shaped antenna gain $G(\theta, \Phi)$ in the θ, Φ direction can be derived from *Equation 10* if we know the antenna gain in the direction of $P_*(\theta_*, \Phi_*)$. Let us refer the shaped antenna gain in the θ_*, Φ_* direction to the gain of an isotropic antenna fed by the total power $\sum \alpha_n = 1$.

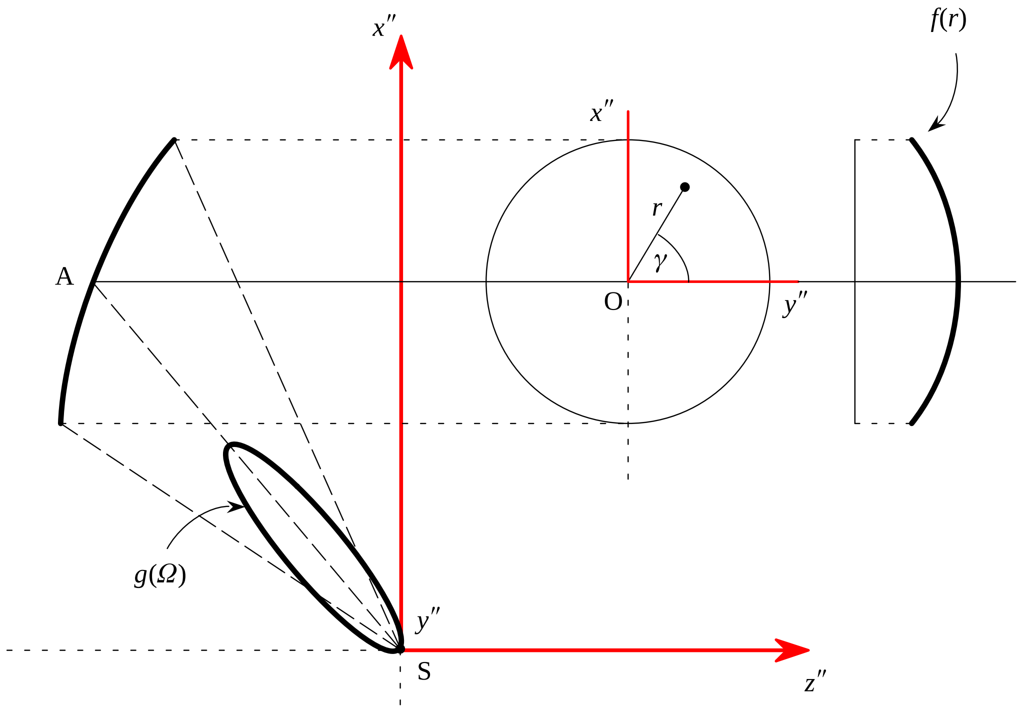


Figure 8

The gain of the shaped antenna in the θ_*, Φ_* direction (see *Fig. 9*) can be evaluated approximately by the formula:

$$10 \log G(\theta_*, \Phi_*) = 10 \log \eta \frac{16}{\Delta\psi^2} \alpha_* + 10 \log \xi$$

in which:

$$\xi = \frac{\left| \sum_n \sqrt{\alpha_n} E_n(\theta_*, \Phi_*) \right|^2}{\alpha_* E_*^2(\theta_*, \Phi_*)}$$

$\Delta\psi$ = the total –3 dB aperture angle (in radians) of the beam radiated by the feed which points at $P_*(\theta_*, \Phi_*)$ (called the "reference feed")

η ≅ 0.55 (the efficiency of the parabolic antenna fed by the reference feed alone)

α_* = the fraction of the total power delivered to the reference feed.

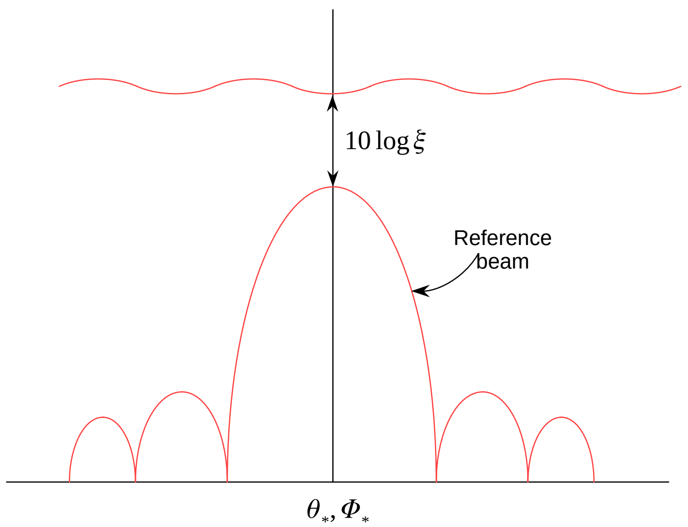
$E_n(\theta_*, \Phi_*)$ and $E_*(\theta_*, \Phi_*)$ are the fields produced in $P_*(\theta_*, \Phi_*)$ by the n-th feed and the reference feed respectively.

Once the gain of the shaped antenna is known in the $P_*(\theta_*, \Phi_*)$ direction (by means of *Equation 10)*, the gain in other directions is easily derived.

6. Computational examples

A comparison between the results obtained by using the method described here and that derived from using TICRA software is presented in *Figs. 10a*, *10b* and *10c*.

Figure 9
The shaped antenna gain in the θ_*, Φ_* direction.



Three different cases have been examined, each using a different illumination function $g(\Omega)$ (see *Fig. 8*). Each of the three cases was based on the following assumptions:

- parabola reflector diameter (D_1) = 3 m
- antenna offset (h_1) = 0.3 m (see *Fig.3*)
- frequency = 12 GHz

In all cases, the minimum f/D_1 ratio was assumed to be about 1. Preliminary computer calculations have in fact shown that this figure can be considered as a lower limit of validity for the model described.

Case 1

A single feed was placed at the parabola focus. The gain roll-off of $g(\Omega)$ in the direction of the reflector edge was equal to –20 dB relative to the maximum gain (i.e. the reflector edge taper = –20 dB) and the focus-to-diameter ratio of the reflector was equal to unity (i.e. $f/D_1 = 1$).

Case 2

Here, there were seven feeds, aligned along the *x* axis[4] of the feeds plane. Each feed had the coordinates shown in the table below.
Reflector edge taper = –5 dB, $f/D_1 = 1.5$.

Feed no.	x (mm)	y (mm)
1	–98.4	0.0
2	–65.6	0.0
3	–32.8	0.0
4	0.0	0.0
5	32.8	0.0
6	65.6	0.0
7	98.4	0.0

Case 3

In this case, there were also seven feeds. However, this time they were aligned along the *y*

Feed no.	x (mm)	y (mm)
1	0.0	–98.4
2	0.0	–65.6
3	0.0	–32.8
4	0.0	0.0
5	0.0	32.8
6	0.0	65.6
7	0.0	98.4

4. The *x* axis on the feeds plane is defined as the intersection of the $x'' - z''$ plane of the reference system $\mathbf{T}''_s$ and the feeds plane (see *Fig. 3*).

axis of the feeds plane and had the coordinates shown in the table above.
Reflector edge taper = –30 dB, f/D_1 = 1.5.

All computations relating to the above cases were produced using TICRA software and a software developed at RAI to run on an IBM-compatible 486 PC (computing time = less than two minutes).

To begin with, the RAI program computes the best values of ϱ and m (see *Appendix 1)* that follow from the chosen gain roll-off value of $g(\Omega)$ in the direction of the reflector edge and the geometrical parameters of the antenna. Then it computes and represents on a U–V reference system (so as to be comparable with TICRA results) the equi-level curves (dB) that represent the radiation pattern of that particular shaped antenna configuration (see *Figs. 10a*, *10b* and *10c*).

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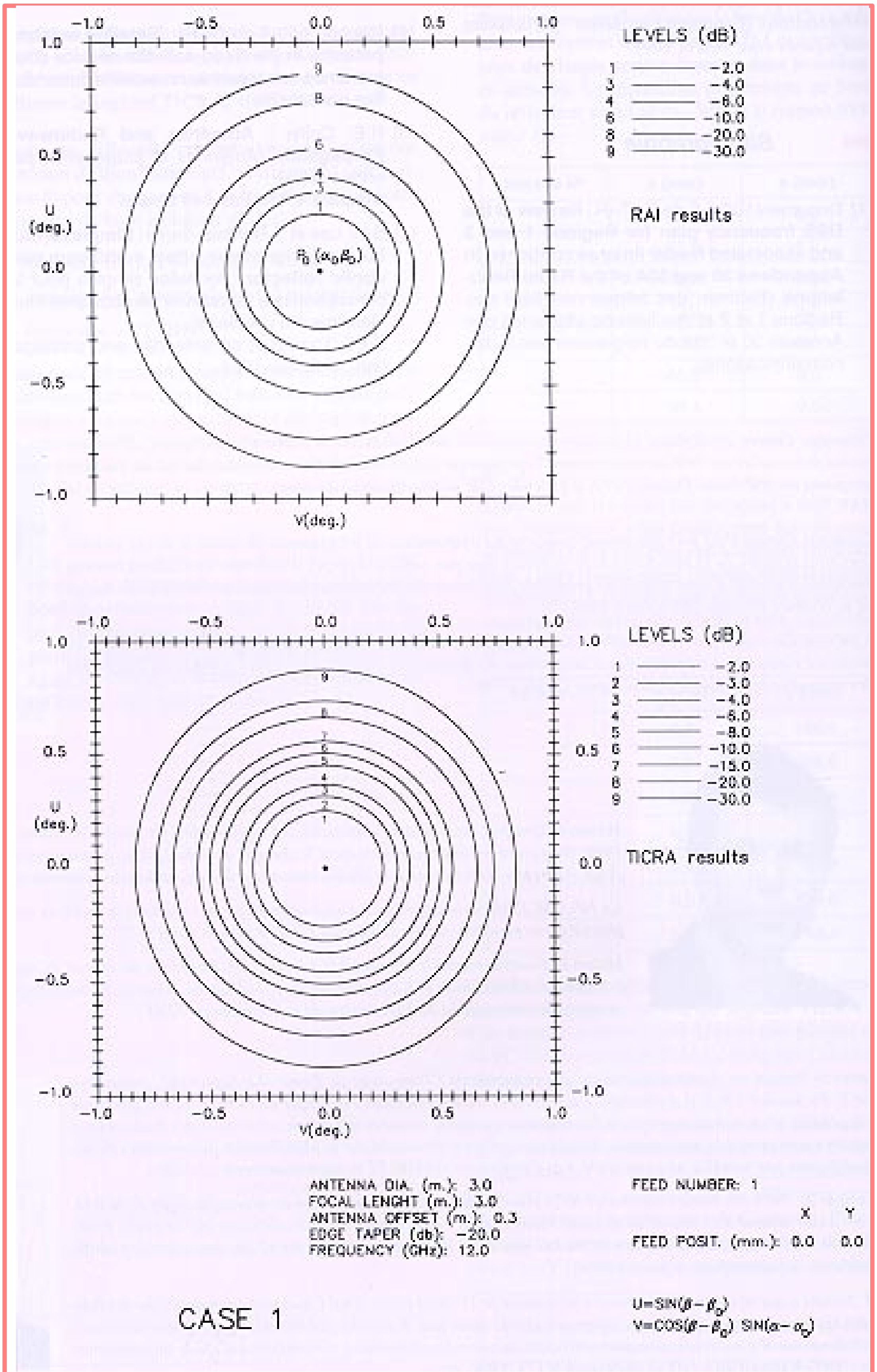


Figure 10a
Antenna radiation patterns: Case 1.

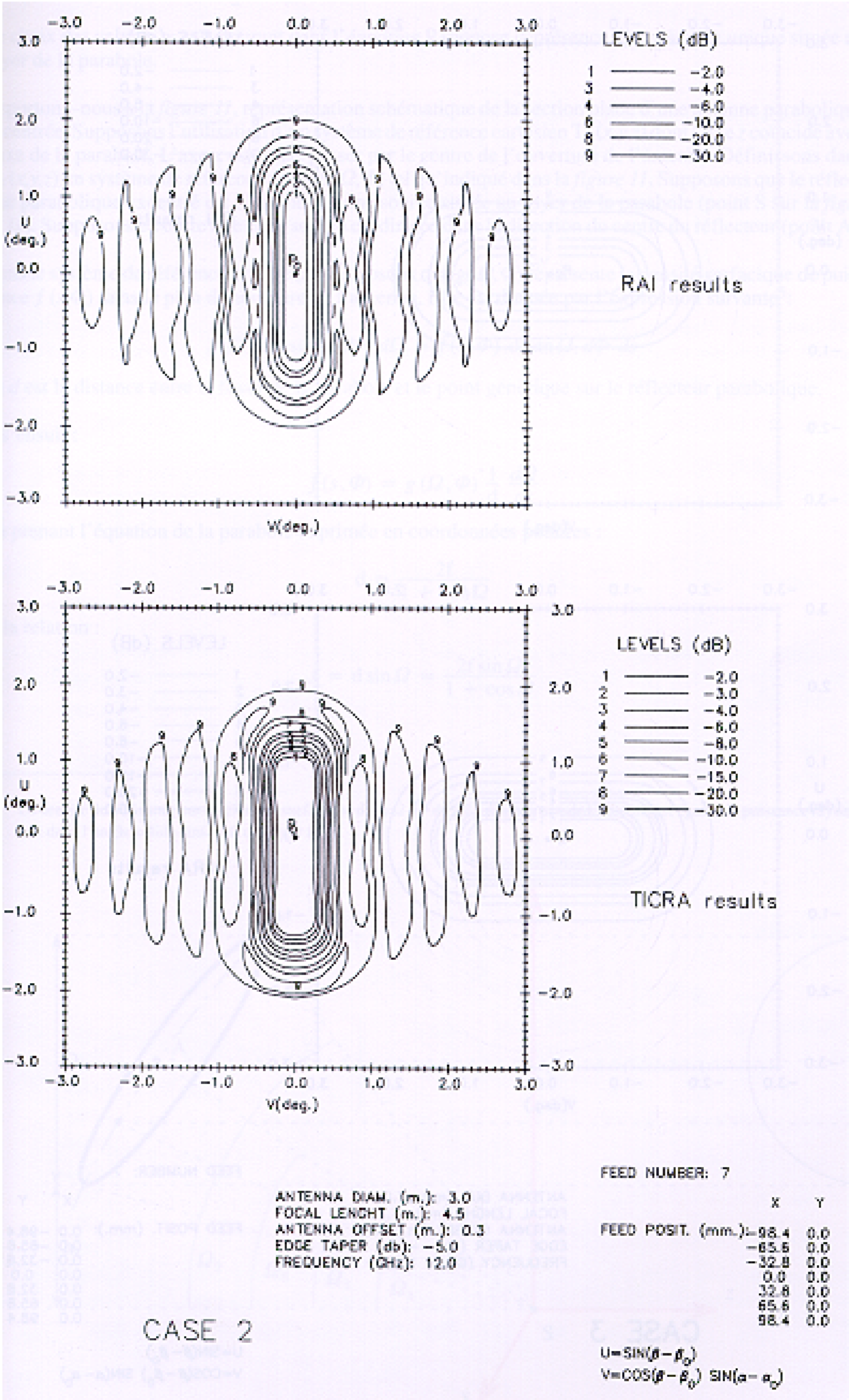


Figure 10b
Antenna radiation patterns: Case 2.

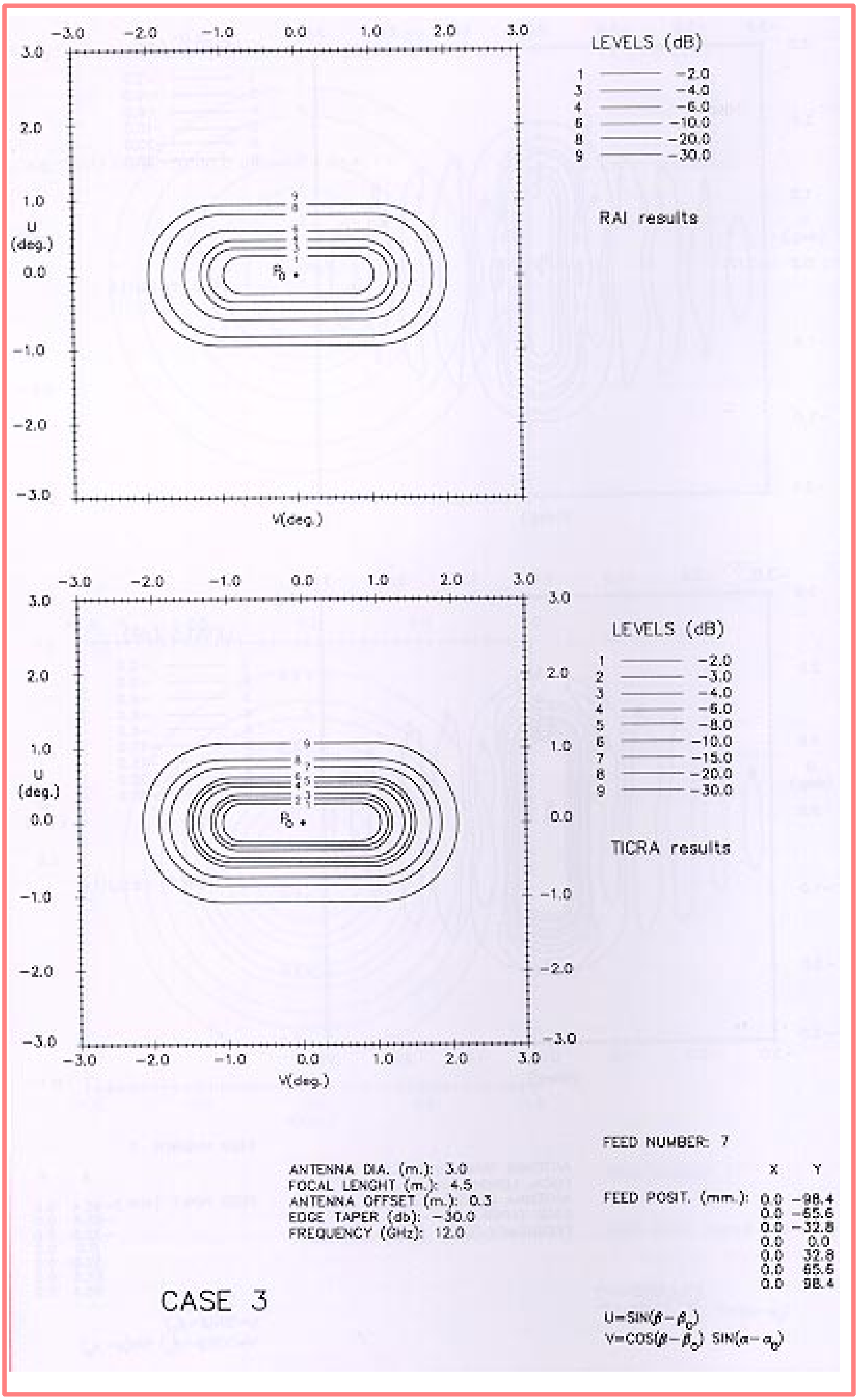


Figure 10c
Antenna radiation patterns: Case 3.

Appendix 1

The choice of the values m and ϱ which are inserted in *Equation 9* is made assuming a single feed placed at the parabola focus.

Let us refer to *Fig. 11* in which a plane section of an offset parabolic antenna is schematically represented. Assume the use of a Cartesian reference system $\mathbf{T}_s(x, y, z)$, having the z axis coincident with the axis of the parabola. The x axis is assumed to pass through the antenna aperture centre. Let us define in $\mathbf{T}_s(x, y, z)$ a reference polar system $[\Omega, \Phi]$ as indicated in *Fig. 11*. Let us assume that the offset parabolic reflector is illuminated by a feed placed in the focus of the parabola (point S on *Fig. 11*); let us further assume that this feed is directed towards the reflector centre (point A).

With reference to the polar reference system $[\Omega, \Phi]$, let $g(\Omega, \Phi)$ represent the power radiation pattern of the feed. The function $g(\Omega, \Phi)$ is related to the surface power density $f(s, \Phi)$ on the antenna aperture plane by the following expression[5]:

$$g(\Omega, \Phi)\ \sin\Omega\ d\Phi\ d\Omega\ =\ f(s, \Phi)\ \mathrm{d}\ \sin\Omega\ d\Phi\ ds$$

where d is the distance between the parabola focus and the generic point on the parabolic reflector.

It follows that:

$$f(s, \Phi)\ =\ g(\Omega, \Phi)\ \frac{1}{\mathrm{d}}\ \frac{d\Omega}{ds}$$

Taking into account the equation of a parabola in polar coordinates:

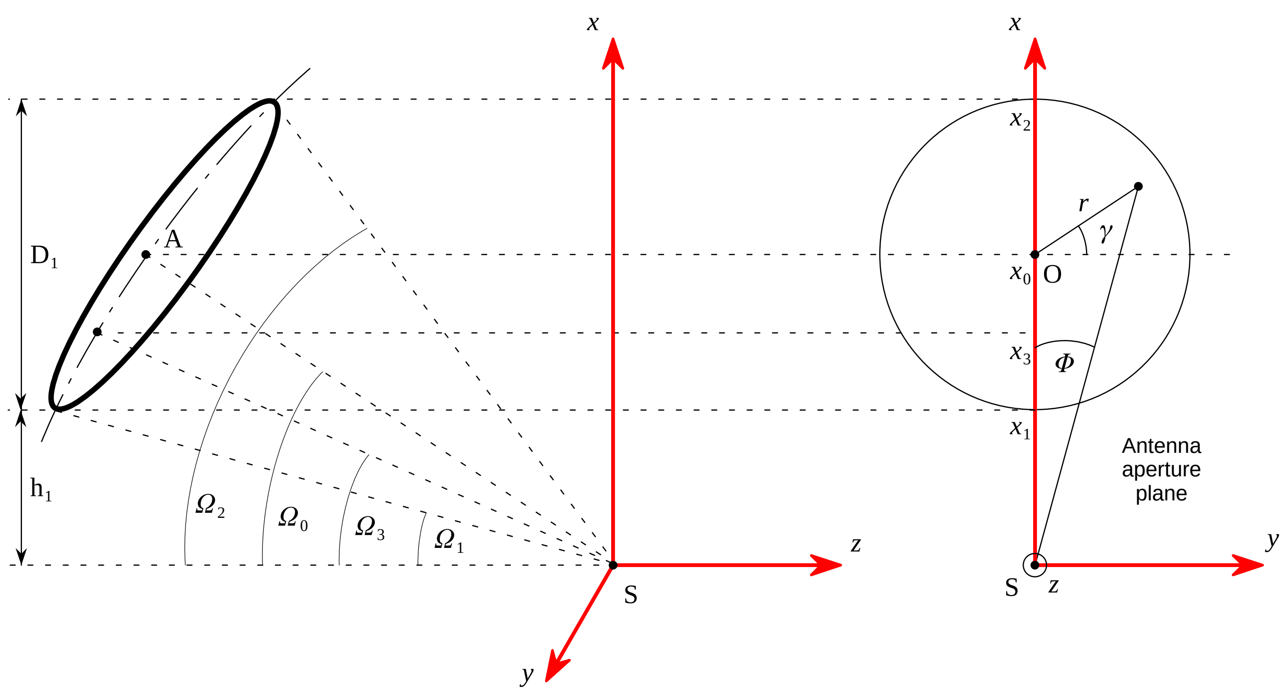
$$\mathrm{d}\ =\ \frac{2\mathrm{f}}{1\ +\ \cos\Omega}$$

and the relationship:

$$s\ =\ \mathrm{d}\sin\Omega\ =\ \frac{2\mathrm{f}\sin\Omega}{1\ +\ \cos\Omega}$$

5. The power density on a surface element d sinΩ dΦ ds of the antenna aperture plane equals the power radiated in the solid angle sinΩ dΦ dΩ (see [3]).

Figure 11



We have:

$$f(s,\Phi) = g(\Omega,\Phi)\left(\frac{1 + \cos\Omega}{2f}\right)^2$$

Let us assume that $g(\Omega,\Phi)$ is circularly symmetric around its maximum value which occurs at $\Omega = \Omega_0$, $\Phi = \Phi_0$ (the polar coordinates of the $\overline{\text{SA}}$ direction).

Normalizing $f(s,\Phi)$ to its value at $\Omega = \Omega_0$, $\Phi = \Phi_0$, we have:

$$f(s,\Phi) = \frac{g(\Omega,\Phi)}{g(\Omega_0,\Phi_0)}\left(\frac{1 + \cos\Omega}{1 + \cos\Omega_0}\right)^2$$

Let us assume a planar polar coordinate system (on the antenna aperture plane) defined by the angle γ and the distance r from the antenna aperture centre *(Fig. 11)*, and let us represent the surface power density $f(s,\Phi)$ in terms of these new coordinates $[r,\gamma]$.

The resulting function $f(r,\gamma)$ is not symmetric around the aperture centre.

In order to simplify calculations we assume that the illuminating function is symmetric around the aperture centre i.e. $f(r,\gamma) = \overline{f(r)}$, and that it has the mathematical form:

$$\overline{f(r)} = 1 - \rho + \rho\left[1 - \left(\frac{2r}{D_1}\right)^2\right]^m \qquad (11)$$

We calculate the coefficients m and ρ in order to bring $f(s,\Phi)$ as close as possible to $\overline{f(r)}$. Towards this end, we equate $f(s,\Phi)$ and $\overline{f(r)}$ at two points on the x axis of the reference system $\mathbf{T}_s(x,y,z)$ (we must not forget that $\overline{f(r)}$ is symmetrical around the antenna aperture centre).

The chosen points (see *Fig. 11*) are those corresponding to:

$$\Omega = \Omega_1 , \ \Phi = \Phi_0 = 0$$

$$\Omega = \frac{(\Omega_0 + \Omega_1)}{2} = \Omega_3 , \ \Phi = \Phi_0 = 0$$

This results in the following equalities:

$$g(\Omega_1)\left(\frac{1 + \cos\Omega_1}{1 + \cos\Omega_0}\right)^2 = 1 - \rho$$

$$g(\Omega_3)\left(\frac{1 + \cos\Omega_3}{1 + \cos\Omega_0}\right)^2 = 1 - \rho + \rho\left\{1 - \left[\frac{2(x_0 - x_3)}{D_1}\right]^2\right\}^m \qquad (12)$$

in which:

$$x_i = 2\,f\,\frac{\sin\Omega_i}{1 + \cos\Omega_i} \qquad (i = 0 , \ i = 3)$$

Suppose we have a feed whose power radiation function $g(\Omega,\Phi)$ (normalized to its maximum value) is Gaussian in distribution around the maximum radiation axis, namely:

$$g(\Omega) = e^{-\alpha(\Omega-\Omega_0)^2} \tag{13}$$

If we replace in *Equation 11* the values calculated by means of *Equation 12*, the graphs shown in *Figs. 12a*, *12b* and *12c* are obtained for different values of the constant α in *Equation 13*. In these graphs, $g(\Omega)$ is characterized by means of its reflector edge taper value (in dB) which is related to α by the expression:

$$\alpha = -\frac{1}{(\Omega_1 - \Omega_0)^2} \ln\left\{10^{\frac{\text{taper (dB)}}{10}}\right\} \tag{14}$$

It follows that the best value of m and ρ depends on the reflector edge taper value of the Gaussian function $g(\Omega)$, as indicated in the figures themselves. Having chosen the required taper values, the best values for m and ρ can be found.

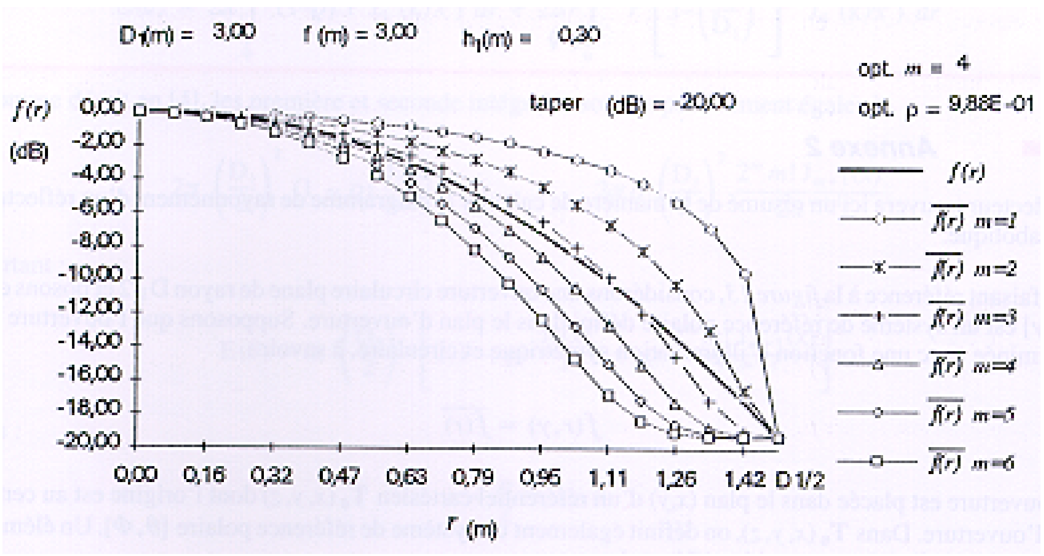


Figure 12a
Relationship between *f(r)* and *r* for different values of *m*: Case 1.

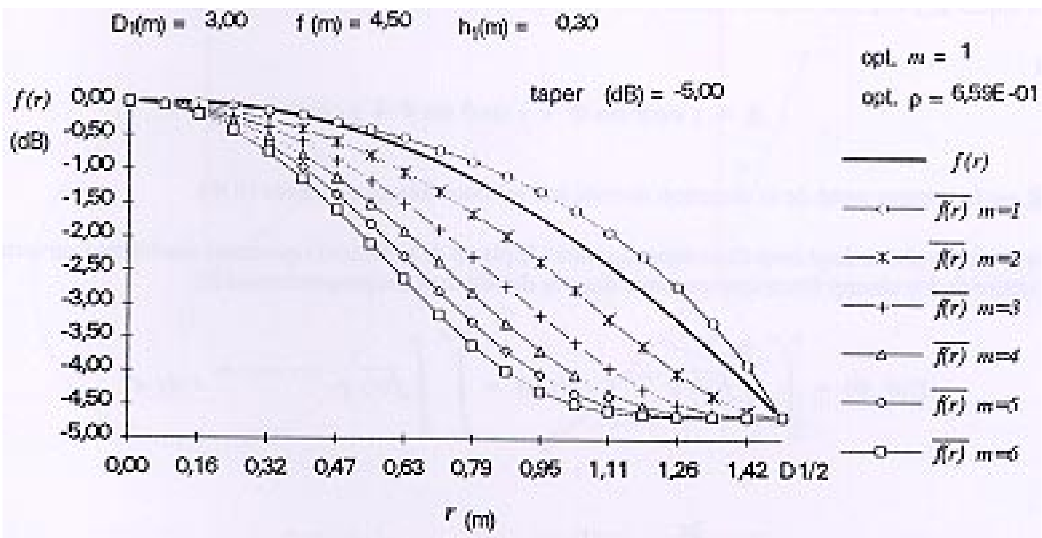


Figure 12b
Relationship between *f(r)* and *r* for different values of *m*: Case 2.

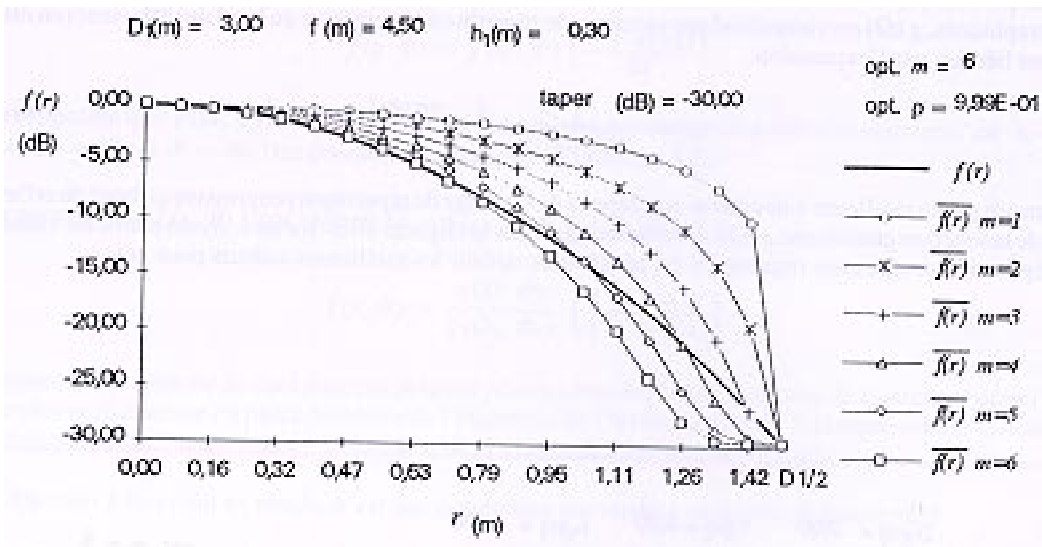


Figure 12c
Relationship between *f(r)* and *r* for different values of *m*: Case 3.

Appendix 2

The derivation of the radiation pattern of a parabolic reflector is summarized here for the benefit of the reader.

With reference to *Fig. 13*, let us consider a plane circular aperture of radius $D_1/2$ and let $[r, \gamma]$ be a polar reference system defined in the aperture plane. Let us assume that the aperture is illuminated with a circular symmetric illuminating function, namely:

$$f(r, \gamma) = \overline{f(r)}$$

The aperture is placed on the (x, y) plane of a Cartesian reference system $\mathbf{T_0}(x, y, z)$ whose origin is at the aperture centre. In $\mathbf{T_0}(x, y, z)$, let us also define a polar reference system $[\theta, \Phi]$. A radiating element of the aperture is identified by the vector:

$$\underline{r} = \underline{x}r \ \cos\gamma + \underline{y}r \ \sin\gamma$$

in which $\underline{x}$, $\underline{y}$ and $\underline{z}$ are, respectively, the unit vectors of the x, y and z axes of $\mathbf{T_0}(x, y, z)$.

Let:

$$\underline{\mathrm{R}} = \underline{x}\sin\theta\cos\Phi + \underline{y}\sin\theta\sin\Phi + \underline{z}\cos\theta$$

where $\underline{\mathrm{R}}$ is the unit vector of the direction identified by the $[\theta, \Phi]$ polar coordinates.

Let us refer the phase of the radiated electric field to the phase of the radiating element situated in the circular aperture O. The electric field radiated in the $\underline{\mathrm{R}}$ direction is proportional to:

$$\mathrm{E}(\theta, \Phi) = \int_0^{\mathrm{D_1}/2} \int_0^{2\pi} \overline{f(r)}\, e^{-\mathrm{jk}\underline{r}\cdot\underline{\mathrm{R}}}\, r\, dy\, dr = \int_0^{\mathrm{D_1}/2} \int_0^{2\pi} \overline{f(r)}\, e^{-\mathrm{jk}rx'\cos(\gamma - \Phi)}\, r\, dy\, dr$$

in which:

$$\mathrm{k} = \frac{2\pi}{\lambda} \qquad \text{and} \qquad x' = \sin\theta$$

By integrating, we get:

$$E(\theta, \Phi) = E(\theta) = 2\pi \int_0^{D_1/2} r\,\overline{f(r)}\,J_0(krx')\,dr$$

The radiation diagram is then symmetric with respect to the axis of $\mathbf{T_0}(x, y, z)$. We assume that:

$$\overline{f(r)} = 1 - \rho + \rho\left[1 - \left(\frac{2r}{D_1}\right)^2\right]^m$$

where the symbols are the same as those defined in *Sections 3* and *5*, and in *Appendix 1*.

It follows:

$$E(\theta) = 2\pi \int_0^{D_1/2} (1-\rho)\,r\,J_0(krx')\,dr + 2\pi r \int_0^{D_1/2} r\left[1 - \left(\frac{2r}{D_1}\right)^2\right]^m J_0(krx')\,dr$$

As described in [4], the first and the second integral, respectively, are equal to:

$$2\pi\left(\frac{D_1}{2}\right)^2(1-\rho)\frac{J_1(x)}{x} \qquad\qquad 2\pi\rho\left(\frac{D_1}{2}\right)^2\frac{2^m m!\,J_{m+1}(x)}{2x^{m+1}}$$

Hence:

$$E(\theta) = \pi\left(\frac{D_1}{2}\right)^2\left[2(1-\rho)\frac{J_1(x)}{x} + \rho\,\frac{2^m m!\,J_{m+1}(x)}{2x^{m+1}}\right]$$

In which:

$$x = \frac{\pi}{\lambda} D_1 \sin\theta$$

Figure 13