Noise and intermodulation in cable distribution networks

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The first in this new series of EBU TUTORIALS looks at the problem of cable television network planning, and more especially the calculation of noise and intermodulation.

1. Calculation of thermal noise

Most cable television distribution networks obtain their programmes from terrestrial television transmitters or from satellites in the fixed satellite service (FSS) or the broadcasting satellite service (BSS). The noise entering the network at the head-end has two components: thermal noise received by the antenna and noise generated in the receiving equipment.

The noise received by the antenna is dependent on the type of antenna and, more especially, on the antenna pattern. *Fig. 1* shows typical antenna patterns for a Yagi antenna, as used for off-air reception from terrestrial transmitters (VHF and UHF), and an antenna for the reception of satellite services, with a parabolic reflector.

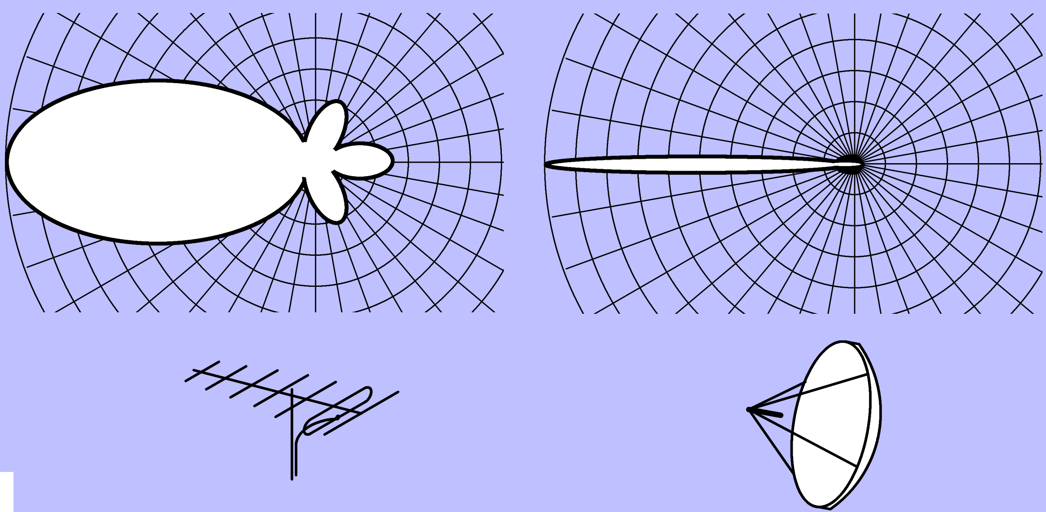

Figure 1
Comparison of vertical antenna patterns.

a) Broad vertical pattern of Yagi antenna.

b) Narrow vertical pattern of parabolic reflector antenna.

The Yagi antenna has a relatively broad vertical antenna pattern (assuming the antenna is installed for the reception of horizontally-polarized transmissions). Therefore the antenna noise temperature of this sort of antenna (or other dipole antennas) depends only very slightly on the elevation angle; in any case, this angle is usually close to zero degrees (horizontal) for reception from terrestrial transmitters. The noise temperature in VHF / UHF reception will therefore be about the same as the absolute temperature of the surroundings. Cosmic noise at these frequencies is at a very low level, so the noise temperature for a Yagi antenna at VHF / UHF will be about 300 K in most situations.

The thermal noise voltage produced by such an antenna may be found using Nyquist's noise formula:

$$U_n = \sqrt{k \times T_a \times B \times R_a} \quad (1)$$

where k = Boltzmann's constant (1.38×10^{-23} Joule/K)
T_a = antenna noise temperature (approx. 300 K in this case)
B = system bandwidth in Hz (4.75×10^6 for the system considered here)
R_a = system impedance (75 Ω).

Substituting these values in equation (1), we obtain:

$$U_n = \sqrt{1.38 \times 10^{-23} \times 300 \times 4.75 \times 10^6 \times 75} = 1.214\ \mu V = 1.7\ dB\mu V \quad (2)$$

The parabolic reflector antenna has a very narrow antenna pattern (*Fig. 1*). As a result, the thermal noise radiation coming into this sort of antenna is very dependent on the elevation angle. In Norway, for example, the elevation angle of a satellite at 5° E on the geostationary orbit ranges from about 9° in the north to 24° in the south. The mean value of antenna noise temperature is about 30 K in such situations [2], and the noise voltage in a 75Ω system is:

$$U_n = \sqrt{1.38 \times 10^{-23} \times 30 \times 4.75 \times 10^6 \times 75} = 0.384\ \mu V = -8.3\ dB\mu V \quad (3)$$

2. Calculation of total noise power

Fig. 2 shows a system comprising an antenna, cables and amplifiers. F is the noise factor, A the attenuation (expressed as a power ratio) and G is the gain (power gain) of each element in the chain. The total noise

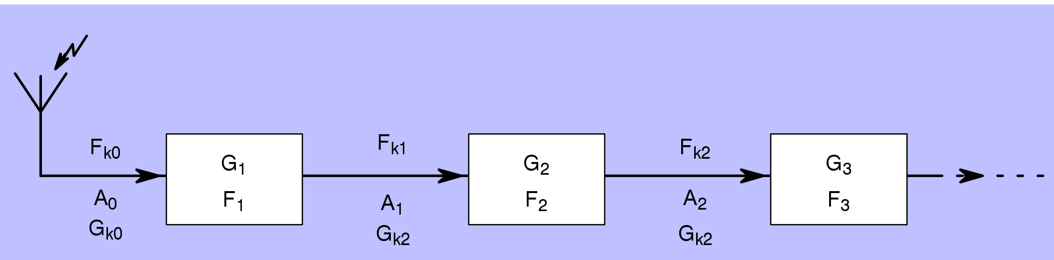

Figure 2
Schematic diagram of a cable distribution system

factor of the system is given by the following formula, often attributed to H.T. Friis:

$$F_{totm} = F_{k0} + \frac{F_1 - 1}{G_{k0}} \quad F_{1tot}$$

$$+ \frac{F_{k1} - 1}{G_{k0} \times G_1} + \frac{F_2 - 1}{G_{k0} \times G_{k1} \times G_1} \quad F_{2tot}$$

$$+ \frac{F_{k2} - 1}{G_{k0} \times G_{k1} \times G_1 \times G_2} + \frac{F_3 - 1}{G_{k0} \times G_{k1} \times G_{k2} \times G_1 \times G_2} \quad F_{3tot}$$

$$+ \; . \; . \; . \; . \quad F_{ntot} \qquad (4)$$

where F_{k0} = noise factor of antenna cable
G_{k0} = gain of cable (less than 1, representing attenuation).

Also, F_1 = noise factor of first amplifier
G_1 = gain of first amplifier
F_{k1} = noise factor of cable between first and second amplifiers
and so on ....

The noise factor of a cable is equal to the attenuation of the cable A_0, so the following equivalences apply:

$$F_{k0} = A_0 = \frac{1}{G_{k0}} \qquad G_{k0} = \frac{1}{A_0}$$

The different parts of equation (4) can be grouped together as shown by the shaded boxes, as a sequence of values F_{1tot}, F_{2tot}, F_{3tot}, etc. This can be used to obtain a more convenient expression for the total noise factor. For the first section of the system, comprising the noise factor of the antenna down-lead (F_{k0}), noise factor of the first amplifier (F_1) and the gain (attenuation) of the down-lead (G_{k0}), we obtain the value of F_{1tot} as follows:

$$F_{1tot} = F_{k0} + \frac{F_1 - 1}{G_{k0}} = F_{k0} + \frac{F_1 - 1}{\frac{1}{A_0}} = A_0 + A_0(F_1 - 1)$$

$$= A_0 \times F_1$$

Using the same procedure of substitution:

$$F_{2tot} = \frac{F_{k1} - 1}{G_{k0} \times G_1} + \frac{F_2 - 1}{G_{k0} \times G_{k1} \times G_1} = \frac{A_0(A_1 - 1)}{G_1} + \frac{A_0 \times A_1(F_2 - 1)}{G_1}$$

$$= \frac{A_0(A_1 \times F_2 - 1)}{G_1}$$

and:

$$F_{3tot} = \frac{F_{k2} - 1}{G_{k0} \times G_{k1} \times G_1 \times G_2} + \frac{F_3 - 1}{G_{k0} \times G_{k1} \times G_{k2} \times G_1 \times G_2}$$

$$= \frac{A_0 \times A_1(A_2 \times F_3 - 1)}{G_1 \times G_2}$$

Then, noting that

$$F_{tot} = F_{1tot} + F_{2tot} + F_{3tot} + ...$$

we obtain:

$$F_{tot} = A_0 \left\{ F_1 + \frac{A_1 \times F_2 - 1}{G_1} + \frac{A_1(A_2 \times F_3 - 1)}{G_1 \times G_2} + \frac{A_1 \times A_2(A_3 \times F_4 - 1)}{G_1 \times G_2 \times G_3} + ... \right\}$$

or:

$$F_{tot}\,(dB) = 10 \log \left\{ A_0 \left\{ F_1 + \frac{A_1 \times F_2 - 1}{G_1} + \frac{A_1(A_2 \times F_3 - 1)}{G_1 \times G_2} + \frac{A_1 \times A_2(A_3 \times F_4 - 1)}{G_1 \times G_2 \times G_3} + .. \right\} \right\} \quad (5)$$

From equation (5) it is seen that the attenuation A_0 before the first amplifier has a decisive influence on the total noise factor. The first amplifier is normally of a low-noise type, however the advantage offered by the use of a low-noise amplifier at the front-end of the system may be lost if there is too much attenuation (and loss) before this amplifier.

Equation (5) also shows that the attenuation in the cable preceding, for example, section 4 of the system (A_3) has a more destructive influence on the contribution to the total noise factor than the cable attenuations closer to the system head-end, preceding the second and third amplifiers (A_1 and A_2).

3. Calculation of total signal-to-noise ratio

If the signal-to-noise ratio, expressed in decibels, for successive sections of a distribution system is denoted by $(S/N)_1$, $(S/N)_2$, $(S/N)_3$, . . . the signal-to-noise ratio obtained at the end of the chain remote from the antenna may be found using the following formula [4]:

$$(S/N)_{tot(dB)} = -10 \log \left\{ 10^{\frac{-(S/N)_{1(dB)}}{10}} + 10^{\frac{-(S/N)_{2(dB)}}{10}} + 10^{\frac{-(S/N)_{3(dB)}}{10}} + \ldots \right\} \quad (6)$$

If all the sections are identical, this formula reduces to:

$$(S/N)_{tot(dB)} = (S/N)_{x(dB)} - 10 \log\ m \quad (7)$$

where x is one of the identical sections
and m is the number of sections.

Equation (7) is an approximation, which is valid if the sections are nearly identical, as may be the case in a well-planned network. In normal practice, however, equation (6) should be used to obtain a more reliable result.

It is possible to use another formula which is simpler than equation (6) but which does not lose any substantial degree of accuracy. If the values of signal-to-noise ratio of the individual sections, expressed in decibels, are symmetrically distributed around a mean value, this mean value may be used instead of the value of $(S/N)_{x(dB)}$ in equation (7). Equation (8) substitutes for $(S/N)x(dB)$ the arithmetic mean of the individual S/N ratios expressed in decibels (i.e. the geometric mean of the S/N values), to give results which will be more accurate in practical situations:

$$(S/N)_{tot(dB)} = \frac{(S/N)_{1(dB)} + (S/N)_{2(dB)} + (S/N)_{3dB} + \ldots}{m} - 10 \log m \qquad (8)$$

4. Calculation of the cross-modulation ratio (inter-modulation ratio)

If the cross-modulation ratio CMR, expressed in decibels, is known for each amplifier in a cable distribution network, the total cross-modulation ratio may be found using the following formula:

$$(CMR)_{tot(dB)} = -20 \log \left\{ 10^{\frac{-CMR_{1(dB)}}{20}} + 10^{\frac{-CMR_{2(dB)}}{20}} + 10^{\frac{-CMR_{3(dB)}}{20}} + \ldots \right\} \qquad (9)$$

If all the sections in the system are identical and have the same output level, equation (9) can be simplified to:

$$CMR_{tot(dB)} = CMR_{x(dB)} - 20 \log\ m \qquad (10)$$

where x is one of the identical sections
and m is the number of sections.

Adopting the same procedure as for equation (8), a new formula can then be obtained for the total cross-modulation ratio:

$$CMR_{tot(dB)} = \frac{CMR_{1(dB)} + CMR_{2(dB)} + CMR_{3(dB)} + \ldots}{m} - 20 \log m \qquad (11)$$

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